Introduction to Bundled Modalities

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NSSLLI 25

Introduction to
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a very informal and light-minded overview

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NASSLLI 25 (assume basics of ML, FOL, model theory of ML)


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There was a song called PPAP

PPAP (Pen Pineapple Apple Pen)

Quantified Predicate Logic

quantifies, variables, predicate, equality and function symbols

$$\forall x \forall y (x + y = y + x), \ \forall x \exists y (x + y = 0)$$

$$\exists x \forall y \neg (y \in x)$$

$$\forall x \exists y \forall z (z \in y \leftrightarrow \forall w (w \in z \to w \in x))$$

Modal Logic

□ (Box): necessity, obligation, forever, knowledge …

◇ (Diamond)： possibity, permission, sometimes ...

Wha … ancée?

Modal Logic

$\Box$ (Box): necessity, obligation, forever …

$\Diamond$ (Diamond): possibity, permission, sometimes ...

Concepts of meta-language brought into objective language

$$\Box A \to A, \; \Box A \to \Box\Box A, \; \Box A \to \Diamond A$$

$$\neg\Box A \to \Box\neg\Box A, \; \Diamond\Box A \to \Box\Diamond A$$

Correspondence between Philo and Math properties

$\Box A \to A$	$\forall x(xRx)$
$\Box A \to \Box\Box A$	$\forall x \forall y \forall z(xRy \wedge yRz \to xRz)$
$A \to \Box\Diamond A$	$\forall x \forall y(xRy \to yRx)$

Do quantifiers and modalities come together often?

Modal Logic ≠ Propositional ML

Quantifiers （∀∃） and modalities （□◊） are equally important


连我都不懂是什么
Even I don't really understand it.

Modal Logic ≠ Propositional ML

Ax. 1.	$(P(\varphi) \wedge \Box \forall x(\varphi(x) \rightarrow \psi(x))) \rightarrow P(\psi)$
Ax. 2.	$P(\neg\varphi) \leftrightarrow \neg P(\varphi)$
Th. 1.	$P(\varphi) \rightarrow \Diamond \exists x\, \varphi(x)$
Df. 1.	$G(x) \leftrightarrow \forall\varphi(P(\varphi) \rightarrow \varphi(x))$
Ax. 3.	$P(G)$
Th. 2.	$\Diamond \exists x\, G(x)$
Df. 2.	$\varphi \text{ ess } x \leftrightarrow \varphi(x) \wedge \forall\psi\, (\psi(x) \rightarrow \Box \forall y(\varphi(y) \rightarrow \psi(y)))$
Ax. 4.	$P(\varphi) \rightarrow \Box P(\varphi)$
Th. 3.	$G(x) \rightarrow G \text{ ess } x$
Df. 3.	$E(x) \leftrightarrow \forall\varphi(\varphi \text{ ess } x \rightarrow \Box \exists y\, \varphi(y))$
Ax. 5.	$P(E)$
Th. 4.	$\Box \exists x\, G(x)$

Modal Logic ≠ Propositional ML

In contrast with what Carnap thought
the history of ML went the other way around

Quantified modal logic seemed to have lots of “problems”

At the same time, research on Prop ML went too well, in particular, balancing expressiveness vs. complexity.

Modal Logic ≠ Propositional ML

The pre-history of bundled modalities

(in a broader sense)

Bundles in existing logics

Polyadic ML

NCL

CTL

WAL

ATL

MNL

…

MEL

Non-contingency logic

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Delta\varphi$$

$\Delta\varphi := \Box\varphi \vee \Box\neg\varphi$ **semantically**

knowing whether, provably decidable…

Computation Tree Logic（CTL）

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid EX\varphi \mid EG\varphi \mid E(\varphi U \varphi)$$

$EG\varphi$: **there is a path on which φ holds forever**

$EF\varphi$ is defined by $E(\top U \varphi)$ but $EG\varphi$ cannot be defined by $E(\cdot U \cdot)$

Alternating-time Temporal Logic (ATL)

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\langle A \rangle\rangle X\varphi \mid \langle\langle A \rangle\rangle G\varphi \mid \langle\langle A \rangle\rangle (\varphi U \varphi)$$

$\langle\langle A \rangle\rangle G\varphi$: there exists a strategy for group A such that A can force φ forever, no matter what others do

Multi-agent Epistemic Logic

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_a\phi \mid S_G\phi \mid E_G\phi$$

$S_G\varphi$: **someone in G knows** φ **(**$\exists x \in G\ K_x\varphi$**)**

Or simply $\bigvee_{i \in G} K_i\varphi$ **if there are finitely many agents**

Monotonic Neighborhood Semantics

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box \phi$$

$\Box\varphi$: **there is** **a neighborhood X, such that all worlds in X satisfy** φ

Coalition logic, Evidence logic…

Polyadic Modal Logic

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Diamond(\varphi, \ldots, \varphi)$$

$\Diamond(\varphi_1, \ldots, \varphi_n)$: **there are** $w_1, \ldots w_n$ **such that** $Rww_1, \ldots w_n$ **and each** w_i **satisfies** φ_i

Weakly Aggregative Logic $\varphi_i = \varphi_1$

Even Propositional Modal Logic

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \Box\varphi$$

A normal modality can be viewed as a bundle in FOL $\Box\varphi := \forall y(xRy \to \varphi^t)$

Guarded fragments and so on…

Modal syllogisms

$$\varphi ::= All(t, t) \mid Some(t, t)$$

$$t ::= A \mid \neg t \mid \Box t$$

Can be viewed as bundles in FOML e.g.,

$$Some(A, \neg \Box B) := \exists x (Ax \land \neg \Box Bx)$$

Bundles are everywhere…

In many cases, the language with bundles can be viewed as fragments of a larger language.

You can play with it… E.g.

$\Box_a \varphi \wedge \Box_b \neg \varphi$

$\neg \varphi \wedge \Box \varphi$

$\varphi \wedge \Box \varphi$

$\Box_a \varphi \wedge \neg \Box_b \varphi$

$\varphi \rightarrow \Box \varphi$

$\Box \Diamond \varphi$

……

$\varphi \wedge \neg \Box \varphi$

There are also general theories

In the past decade, we have been playing with specific bundles packing a quantifier and a modality together

Intentionally use them as conceptual and technical tools

Discover various hidden ones


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We will be looking at some examples

Epistemics Logic of Know-wh

Intuitionistic and Intermediate Logics

Deontic Logic

Bundled Fragments of First-order Modal Logic

Each topic can be a separate course but we will focus on the core ideas behind techniques

Epistemic Logic

traditionally focuses on know-that

Know that A = rule out $\neg A$ epistemic possibilities

$$K(A \to B) \to (KA \to KB)$$

$$KA \to A$$

$$(KA \land KB) \to K(A \land B)$$

$$KA \to KKA, \neg KA \to K\neg KA$$

Epistemic Logic

Beyond “knowing that”

Know how/why/what/who/when …

How to do epistemic reasoning
about Know-wh (know+embedded question)?


$\Box A$: there is a method, I know that it can guarantee A …
$\exists x \Box$ (*de re*) vs. $\Box \exists x$ (*de dicto*)

Not just mention-some
but also mention-all
and much more

Kv, Kh, Ky, Kwho…

E.g., knowing who came to the party

for each relevant person, I know whether she/he came to the party

Check SEP entry on EL for a survey

Kv, Kh, Ky, Kwho…

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}p \land \mathcal{K}(p \to q) \to \mathcal{K}q$	NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$
T	$\mathcal{K}p \to p$	EQREPKh	$\dfrac{\varphi \to \psi}{\mathcal{K}h\varphi \to \mathcal{K}h\psi}$
4	$\mathcal{K}p \to \mathcal{K}\mathcal{K}p$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg\mathcal{K}p \to \mathcal{K}\neg\mathcal{K}p$		
AxKtoKh	$\mathcal{K}p \to \mathcal{K}hp$		
AxKhtoKhK	$\mathcal{K}hp \to \mathcal{K}h\mathcal{K}p$		
AxKhtoKKh	$\mathcal{K}hp \to \mathcal{K}\mathcal{K}hp$		
AxKhKh	$\mathcal{K}h\mathcal{K}hp \to \mathcal{K}hp$		
AxKhbot	$\neg\mathcal{K}h\bot$		

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≈
∃, ∀
Pxy
□, ◊
Khφ → KKhφ
KhKhφ → Khφ
KKhφ → KhKφ
Kφ → Khφ

Typical difficulties

Simple language

Rich model

Hard to axiomatized !

We will use logic of know-value to demonstrate the techniques

It is just the beginning…

Non-classical logic

Intuitionistic logic and its relatives


$p \strictif q$
$\Box(p \to q)$


€0,44
⊭ A ∨ ¬A


NEDERLAND
€0,44
$\nvdash A \vee \neg A$
30


DEL
Kripke Semantics
BHK
Uniformity
Intellectualist know-how
∃□
Modal embedding
Hintikka Knowledge of things
Heyting's early writings
Medvedev's problem semantics


€0,44
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⊭ A ∨ ¬A


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⊭A∨¬A

System SDELKh

Axioms

TAUT	Propositional tautologies	KhK	$\mathcal{K}h\alpha \to \mathcal{K}\alpha$
DISTK	$\mathcal{K}(\varphi \to \psi) \to (\mathcal{K}\varphi \to \mathcal{K}\psi)$	KKhp	$\mathcal{K}p \leftrightarrow \mathcal{K}hp$
T	$\mathcal{K}\varphi \to \varphi$	Kh$\bot$	$\mathcal{K}h\bot \leftrightarrow \bot$
4	$\mathcal{K}\varphi \to \mathcal{K}\mathcal{K}\varphi$	Kh$\vee$	$\mathcal{K}h(\alpha \vee \beta) \leftrightarrow \mathcal{K}h\alpha \vee \mathcal{K}h\beta$
5	$\neg\mathcal{K}\varphi \to \mathcal{K}\neg\mathcal{K}\varphi$	Kh$\wedge$	$\mathcal{K}h(\alpha \wedge \beta) \leftrightarrow \mathcal{K}h\alpha \wedge \mathcal{K}h\beta$
DIST$_\Box$	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$	Kh$\to$	$\mathcal{K}h(\alpha \to \beta) \leftrightarrow \mathcal{K}\Box(\mathcal{K}h\alpha \to \mathcal{K}h\beta)$
T$_\Box$	$\Box\varphi \to \varphi$	4_{Kh}	$\mathcal{K}h\alpha \to \mathcal{K}\mathcal{K}h\alpha$
4$_\Box$	$\Box\varphi \to \Box\Box\varphi$	5_{Kh}	$\neg\mathcal{K}h\alpha \to \mathcal{K}\neg\mathcal{K}h\alpha$
PR	$\mathcal{K}\Box\varphi \to \Box\mathcal{K}\varphi$	EU$_k$	$\alpha \wedge \bigwedge_{1\le i\le k}\widehat{\mathcal{K}}(\alpha \wedge \alpha_i) \to \Diamond(\mathcal{K}\alpha \wedge \bigwedge_{1\le i\le k}\widehat{\mathcal{K}}\alpha_i)$
Per	$\alpha \to \Box\alpha$		($k \in \mathbb{N}, \alpha_i \in$ PL for $i \in \mathbb{N}$)
Ver	$\alpha \to \Diamond\mathcal{K}h\alpha$		
		where	$\alpha \in$ PL, $p \in$ P, $\varphi \in$ DELKh

Rules:

MP $\dfrac{\varphi,\ \varphi \to \psi}{\psi}$ NECK $\dfrac{\vdash \varphi}{\vdash \mathcal{K}\varphi}$ NEC$_\Box$ $\dfrac{\vdash \varphi}{\vdash \Box\varphi}$

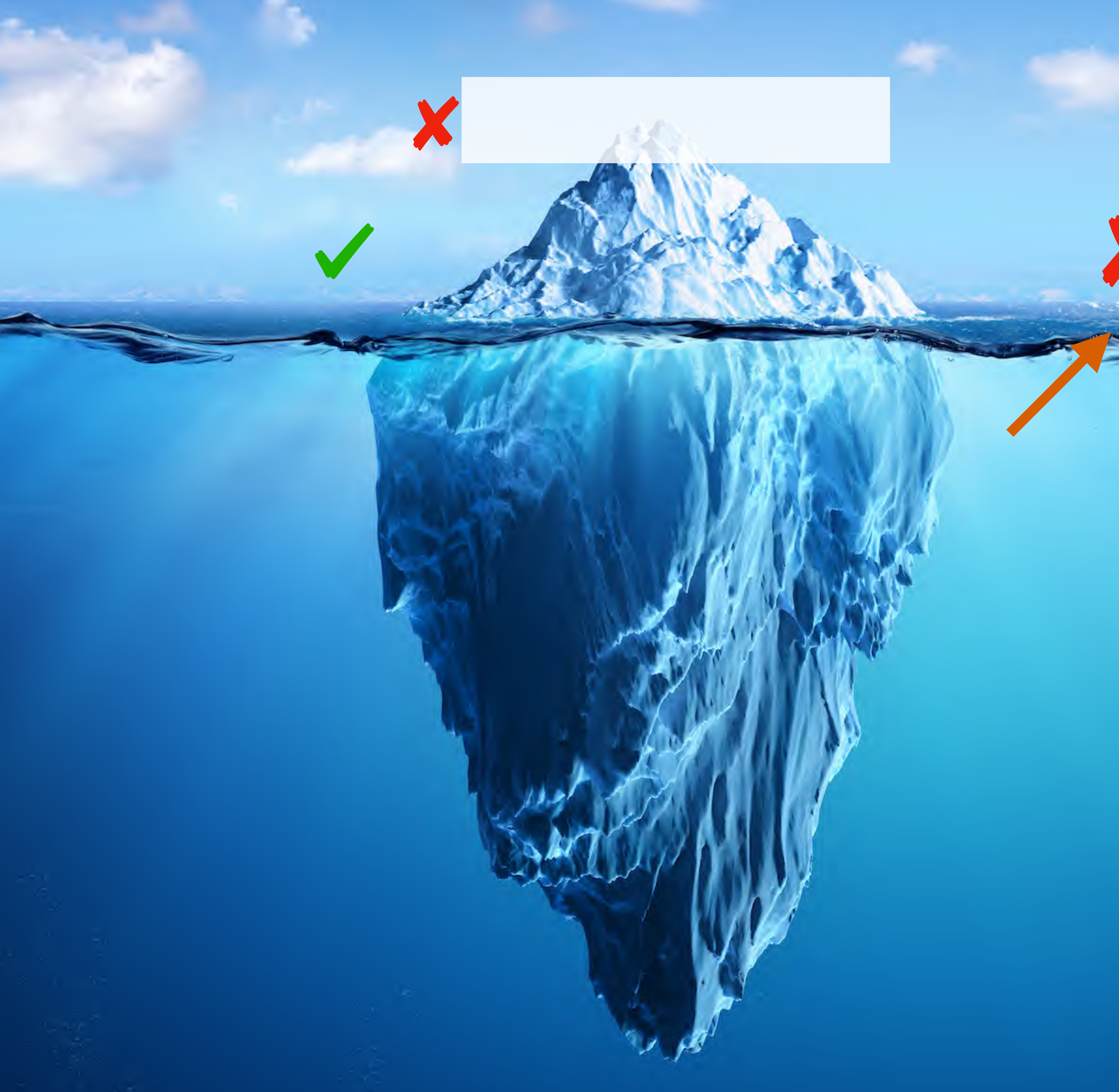
Deontic Logic

Obligation (O), Permission (P), Forbidden (F)

$$OA \rightarrow PA \qquad OA \leftrightarrow \neg P \neg A$$

$$\neg PA \rightarrow FA$$

Lots of interesting puzzles!



Axioms

TAUT	Propositional Tautologies
FC	$\mathbf{P}(\alpha \vee \beta) \leftrightarrow (\mathbf{P}\alpha \wedge \mathbf{P}\beta)$
$\text{COM}_\wedge$	$\mathbf{P}(\alpha \wedge \beta) \leftrightarrow \mathbf{P}(\beta \wedge \alpha)$
$\text{ASSO}_\wedge$	$\mathbf{P}((\alpha \wedge \beta) \wedge \gamma) \leftrightarrow \mathbf{P}(\alpha \wedge (\beta \wedge \gamma))$
DIST	$\mathbf{P}(\alpha \wedge (\beta \vee \gamma)) \leftrightarrow \mathbf{P}((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$
CE	$\mathbf{P}(\alpha \wedge \beta) \rightarrow (\mathbf{P}\alpha \wedge \mathbf{P}\beta)$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Can we build a more abstract logical foundation for these bundles?

We can look at them in first-order modal setting.

First-order modal logic

$$\varphi ::= P\vec{x} \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x\varphi \mid \Box\varphi$$

Hard to find decidable fragments, even only with two variables and unary predicates…

monodic fragment: only allow **one** x in $\Box$

Bundled fragments
of first-order modal logic, e.g.

$$\varphi ::= P\vec{x} \mid \neg\varphi \mid (\varphi \land \varphi) \mid \exists x \Box\, \varphi$$

decidable (PSPACE), finite tree model property, natural characterization…

No restriction on number of variables, arity of predicates, and how many x occur in $\Box$

More bundles

∀□ **AB**

∃◇

□∃ **BE**

◇∀

∀◇

∃□ **EB**

□∀ **BA**

◇∃

∀□ + □∀ **ABBA…**

An (almost) complete picture

Domain	$\forall\Box$	$\exists\Box$	$\Box\forall$	$\Box\exists$	Upper/ Lower Bound
Constant	✓	⋆	⋆	⋆	Undecidable
	⋆	⋆	✓	⋆	
	✗	✓	✗	✗	PSPACE-complete
	✗	✗	✗	✓	No FMP
	✗	✓	✗	✓	
Increasing	✓	✗	✗	✗	PSPACE-complete
	✗	✓	✗	✗	
	✗	✗	✓	✗	
	✗	✗	✗	✓	EXPSPACE/ PSPACE
	✓	✓	✗	✗	EXPSPACE/NEXPTIME
	✗	✗	✓	✓	
	⋆	✓	✓	⋆	Undecidable
	✗	✓	✗	✓	No FMP
	✓	✓	✗	✓	Undecidable
	✓	✗	✓	✓	EXPSPACE/ NEXPTIME
	loosely bundled				

Repeat the secret of propositional modal logic:

restricting the power of quantifiers, but using modalities

Summary:

We focus on bundles packing a quantifier and a modality together

They occur implicitly in logical structure underneath natural language expressions

They may reveal hidden structure which can explain the behaviors of non-classical logics and non-normal modal logic

Bundles often make computation cheaper (as sweet spots)

Hope to promote more research on (fragments of) quantified modal logic

Now you know that bundles are useful.

The next days: let you know how and why

Plan:

Epistemic logic of logic-wh and basic techniques (logics of know-value as a minimal example)

Epistemic interpretation of intermediate logic based on bundles

Bundled approach to deontic logic

Bundled fragments of FOML

WEB BANNER TEMPLATE

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