



Advanced Modal Logic

Conclusions, Reflections, and Further Pointers

Yanjing Wang

Department of Philosophy, Peking University

wangyanjing.com

May 28th, 2024



钻石与盒子的故事

Conclusions, Reflections, and Further Pointers

Yanjing Wang

Department of Philosophy, Peking University

wangyanjing.com

May 28th, 2024

What is modal logic?

Important Ideas

Further readings

Reflections

What is modal logic?

Back to the original questions

- Modal logic is non-classical.
- Modal logic is the logic of “necessity” and “possibility”.
- Modal logic is obtained from propositional logic by adding a modality operator \Box .
- Modal logic is a fragment of first-order logic/MSO.
- Modal logic is useful in many application areas in CS.
- Modal logic is in PSPACE.

Features in each component of $(\mathbf{L}, \mathbf{C}, \mathbf{F}, \mathbf{F})$. It has many good properties: robust decidability, finite tree model property, compactness, interpolation, neat characterization ...

Modal Logic as an *umbrella term* for logic studies related to modalities (and possible worlds). Modal logic is *flexible*.

(Basic normal) Modal logic is:

- The bisimulation invariant fragment of FOL **over models**
- a mixture of FO and MSO **over frames**
- The maximal compact logic invariant under bisimulation
- Boolean Algebra with Operators
- PSPACE-complete (SAT problem)

Conceptually...

Technicality is not the whole point.

- Taking concepts in the **meta-language** into the object-language.
- Formalizing **philosophical** theories just as mathematical theories, but with **semantic insights** as well.
- **Flexible** with language, semantics and proof systems.
- **Balancing** complexity and expressiveness.
- **Bridging** philosophy, mathematics and computer science.

The **charm** of modal logic lies in its **concrete incarnations**, which we did talk much...

Important Ideas

Proof ideas: 曲线救国 (detour), 从小看老 (construct ultrafilter), 里应外合 (closure for definable class), 一石激起千层浪 (satisfiable theory), 破镜重圆 (disjoint union of generated submodels), 画地为牢 (definability results), 游戏人生 (games), 左右互搏 (Bisimulation), 深入浅出 (strengthen the hypothesis), 以点代面 (Jankov-Fine), 以偏概全 (Sahlqvist, incompleteness), 警报范围 (Otto's proof), 循序渐进 (Lindenbaum), 天天向上 (step by step), 改头换面 + 添砖贴瓦 (transforming canonical models), 铺陈展开 (Unraveling bulldozing), 追根溯源 (Kracht), 基缘不和 (McKinsey FO-undefinability), 有穷片段 (Rosen), 表达完备 (Lindström), 形意变换 ((in)completeness), 潜在公式 (ultrafilter extension), 几乎所有 (ultraproduct), 大姐替二姐 (Sahlqvist diamond), 框架换模型 (Goldblatt-Thomason), 表达变区分 (van Benthem), 函数改关系 (bisimulation), 以小见大 (filtration), 以大见小 (validity preservation), 一言以蔽之 (characterizing formula), 树比格子好 (decidability/complexity), 时空转换 (PSPACE), 余是补的归纳 (coinduction), 具体表抽象 (representation theorems), 言无尽而意有穷 (finite many formulas/types modulo...)

Some conceptual points (beyond modal logic)

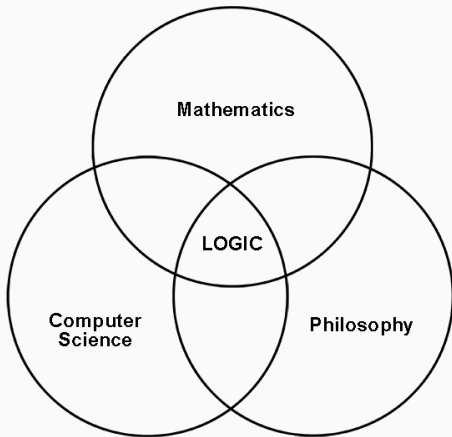
- The notion of sameness is important
- Axioms and premises are different: modal theories/logics
- Viewing the same thing from different perspectives helps.
- To really understand an apple, you probably need a pear.
- A notion of fruits help you to see more ($\mathbf{L}, \mathbf{C}, \vDash, \vdash$).
- Design a looking glass to focus on the things you care.
- Intuitive semantics leads to applications.
- More constraints push you to understand deeper.
- Recognize the importance of **ideas** behind definitions, theorem statements, and connections between things.
- Define things based on how you plan to use them.

Some conceptual points

- Negative results also demonstrate importance of logic.
- Infinite objects sometimes better behave than finite ones.
- Syntax vs. Semantics, \exists vs. \forall , finite vs. infinite.
- Use a logical framework: model checking, automated reasoning, consistent checking, conceptual modelling...
- The current driving force of new developments of logic often comes from TCS and AI.
- Jump out of the box and ask *why not*.
- Have an open mind but don't drop your brain....
- Technicality should serve for ideas.
- The connections between things depend on how you look at them.

Maybe some years later you will understand my points better.

做个三体人: know whether, know why, know how...



“哲学的数学, 数学的哲学” (魏宇)

Further readings






What we did not cover

- Non-classical logics based on Kripke-like semantics
 - Intuitionistic logic and intermediate logics in general
 - Conditional logics
 - Relevant logic...
- Other general semantics
 - Neighbourhood Semantics
 - Coalgebraic modal logic
- Extensions
 - Modal μ -calculus
 - Multi-dimensional modal logic
 - ...

What we did not cover

- Proof theory of modal logic
- First-order modal logic
- ML with propositional quantifiers
- Meta structure of family of logics
- ML of specific domains, e.g., temporal logic (CS), epistemic logic, deontic logic, provability logic...
- Applications: description logic, formal semantics

Further pointers (books) i

-  Johan van Benthem
Modal Logic for Open Minds. CSLI, 2010.
-  Patrick Blackburn, Johan van Benthem, Frank Wolter (eds.)
Handbook for Modal Logic . Elsevier, 2006.
-  D. M. Gabbay, F. Guenther (eds.)
Handbook of Philosophical Logic Springer
-  M.J. Cresswell, G.E. Hughes
A New Introduction to Modal Logic. Routledge, 1996
-  Melvin Fitting and Richard L. Mendelsohn
First-Order Modal Logic, Kluwer Academic Publishers, 1998.

Further pointers (books) ii



James Garson

Modal Logic for Philosophers. Cambridge University Press, 2006.



Brian Chellas

Modal Logic: An Introduction. Cambridge University Press, 1980.



Marcus Kracht





Tools and Techniques in Modal Logic, Elsevier, 1999.



Alexander Chagrov, Michael Zakharyashev

Modal Logic, Clarendon Press, 1997.

Further pointers (books) iii

-  A. Kurucz F. Wolter M. Zakharyashev Dov M. Gabbay
Many-Dimensional Modal Logics: Theory and Applications
Elesvier, 2003
-  Timothy Williamson
Modal Logic as Metaphysics. Oxford University Press, 2013
-  Richard Zach
Boxes and Diamonds: An Open Introduction to Modal Logic. Open Logic Project, 2020
-  文学锋
模态逻辑教程. 科学出版社, 2021

To know the recent developments

The academic delay...

- Look at papers in *Advances in Modal Logic*
(<https://dblp.org/db/conf/aiml/index.html>)
- Subscribe to the google scholar alert with key words such as “modal logic”
- Others conferences and journals (alphabetically):
 - Conferences: AAI, AAMAS, CSL, DEON, DL, IJCAI, KR, LICS, LOFT, LORI, MFCS, TARK...
 - Journals: ACM Transactions on Computational Logic, Annals of Pure and Applied Logic, Artificial Intelligence, Journal of Logic and Computation, Journal of Philosophical Logic, Journal of Symbolic Logic, Logic Journal of IGPL, Notre Dame Journal of Formal Logic, Review of Symbolic Logic, Synthese, Studia Logica ...

Reflections

Review the goals of the course

知识储备: 知道正规模态逻辑的基本结果和重要定理的证明策略, 以及一些关键定义的发现过程, 大概知道“两条线”。

逻辑技能: 对于新的(模态)逻辑语言和语义知道可以问什么技术问题, 能解决一些问题, 至少对问题的难易程度及关键点有正确的评估, 形成一定的“数学直观”。有基础技术才能用起来。

学术技能: 熟练阅读英文教材及文献, 知道如何搜索、下载文献及相关资料。能用 LaTeX 写作业做 slides (可用 overleaf, texpage 入门), 能与人合作做学术报告。

态度培养: 多问“为什么?”形成一种看问题的“模态视角”。不惧复杂问题, 认真细致, 虚心但不盲从, 有一个开放心态。愿意用逻辑工具形式化地分析问题, 学以致用。快慢不那么重要, 但要尽可能深刻, 不怕犯错, 争取不断进步。逐渐形成自己的学术品位, 不仅做一个欣赏者, 也要有志于做一个有问题意识的创造者。

Reflections about the course

- Live vs. Recordings
- P vs. NP
- Textbooks vs. research papers
- Covering all vs. being selective
- Understanding vs. explaining (to others)
- Doing homework vs. doing research
- Student pre. vs. teaching

Let me know your opinion.

Next semester: **Epistemic logic** (Wed. 15:10-18:00)

- You can use your knowledge of modal logic to study a formal account of knowledge (in a multi-agent setting).
- and its application in Philosophy, TCS, AI, Game theory
- Beyond the propositional modal logic:
 - Knowing what/how/why/what/who...
 - Novel decidable fragments of First-order modal logic
 - **Decoding** a family of non-classical logics: intuitionistic logic, Medvedev logic, inquisitive logic, dependence logic, truth maker semantics and various hyperintensional logics.
 - Solving puzzles in deontic logic and other fields.
- A concrete gateway to **many branches** of logic.
- Understand better the world around you.
- Learning it by doing it!

Acknowledgements

Thanks to:

- TA: Wen Tang
- You guys: for your patience and *hard work!*

Modal logic is still a very young field with lots of opportunities (especially first-order modal logic and polyadic modal logic)

