Epistemic Logic (VIII)

Beyond "knowing that": Introduction

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Beyond "knowing that"

Knowledge is not only expressed in terms of "knowing that":

- I *know whether* the claim is true.
- I *know what* your password is.
- I *know how* to go to the hotel.
- I *know why* he was late.
- I *know who* proved this theorem.
- I don't know how to win the game but I know that she knows how and I know why she knows.

Hits (in millions) returned by google:

X	that	whether	what	how	who	why
"know X"	574	28	592	490	112	113
"knows X"	50.7	0.51	61.4	86.3	8.48	3.55

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Linguistically: factivity, exhaustivity, concealed questions

Philosophically: reducible to "knowledge-that"?

Logically: **how to reason about "knowing-wh"?**

Computationally: efficient representation and reasoning

We indeed want to know why /how /what....

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Rated Inspiring, Informative

Emma Marris
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Eric Haseltine
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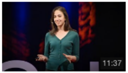

Julia Galef
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Brian Little
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How to fix a broken education system ... without any more money
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Keolu Fox
Why genetic research must be more diverse
Posted Jun 2016
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Gill Hicks
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It helps to go back to the starting point of epistemic logic.

Beyond "knowing that": Hintikka's early work

"knowing who" was discussed by Hintikka (1962) in terms of first-order modal logic: $\exists x\mathcal{K}(Mary \approx x)$, i.e., knowing the answer of the embedded question.

Hintikka used epistemic logic to understand questions. E.g, consider the question Q : "Who murdered Mary?"

- The *presupposition* of Q is $\mathcal{K}\exists xM(x, Mary)$.
- The *desideratum* of Q is $\exists x\mathcal{K}M(x, Mary)$.
- One possible answer to Q is $M(John, Mary)$.
- *Conclusiveness* of the answer requires $\exists x\mathcal{K}(John \approx x)$.
- Conclusive answers realize the desideratum ($\mathcal{K}\exists x$ to $\exists x\mathcal{K}$).

The logic tool for knowing-wh

knowledge-that	—	propositional modal logic
knowledge-wh	—	first-order modal logic

In *Meaning and Necessity* (1947), Carnap remarked:

> *Any system of modal logic without quantification is of interest only as a basis for a wider system including quantification. If such a wider system were found to be impossible, logicians would probably abandon modal logic entirely.*

However, it seems that history went exactly the other way around.

Many things can be done in first-order modal logic

First-order modal logic is infamous for:

- issues in the semantics
- *quantifying-in* and substitution
- ambiguity: *de re* vs. *de dicto*
- incompleteness
- lack of Craig's interpolation
- undecidablility (hard to find useful decidable fragments)
- ....

At the same time, propositional modal logic is too successful...

Forgotten gem?

The early scattered discussions on know-wh seem to be largely forgotten in the later literature, for example:

- In the latest Handbook of Epistemic Logic (2015), there is hardly anything explicitly about first-order epistemic logic nor logic of know-wh.
- In the very same paper where public announcement logic was proposed, Plaza (1989) actually spent half of the paper discussing know-what (the value is).
- The operator was discussed earlier by Xiwen Ma and Weide Guo from Peking University (IJCAI 83).

“Classic” - a book which people praise and don’t read.

– Mark Twain

Recent developments for FO epistemic logic

A slightly out-dated survey in Gochet and Gribomont (2006)

Mostly application-driven (not an exhaustive list):

- about games: Kaneko and Nagashima (1996)
- about cryptographic knowledge: Cohen and Dam (2007)
- about security protocols: Belardinelli and Lomuscio (2011)
- (un)decidability: Wolter (2000), Sturm et al (2000)
- *de dicto* vs. *de re*: distinction Corsi and Orlandelli (2011)
- "second-order" epistemic logic: Belardinelli and van der Hoek (2015, 2016)
- ...

Beyond knowing that: starting point

As philosophical logicians, we design specific-purpose languages to stay at the appropriate abstraction level to highlight the concepts in concern.

Instead of using the full language of first-order modal logic, we can use some well-behaved *fragments* of it to focus on what we really care but no more.

Can we repeat the success of propositional modal logic by a systematic approach to know-wh?

- simple language
- intuitive semantics
- useful models
- balanced expressive power and complexity...

The minimalist's "bundle" approach [Wang18]

- take a know-wh construction as a single modality (a "bundle"), e.g., pack $\exists x \mathcal{K}(Mary \approx x)$ into *Kwho Mary*
- the use of quantifiers is restricted (recall the secret of success of propositional modal logic).
- natural and succinct to express the desired properties, e.g., I *know that* you *know what* the password is but I do not know the password.
- capture the essence of the relevant reasoning by axioms.
- lead to new decidable fragments of first-order modal logic.
- lead to intuitive understanding of non-classical logics.
- stay (technically) neutral for certain philosophical issues.

For each know-wh: the general steps

- focus on some (logically) interesting interpretation
- give natural semantics guided by the first-order modal formulation and linguistic/philosophical theories;
- axiomatize logics with (combinations of) new operators;
- simplify the semantics while keeping the validities;
- capture the expressivity via notions of bisimulation;
- dynamify those logics with new updates of knowledge;
- automate the inferences based on decidability;
- come back to philosophy and linguistics with new insights and questions.

Beyond knowing that: (technical) difficulties

- (apparently) not normal:
  - $\nvdash Kw(p \to q) \land Kw\, p \to Kw\, q$
  - $\nvdash Kh\varphi \land Kh\psi \to Kh(\varphi \land \psi)$
  - $\vdash \varphi \not\Rightarrow \vdash Ky\varphi$
- not strictly weaker: $\vdash Kw\varphi \leftrightarrow Kw\neg\varphi$;
- combinations of quantifiers and modalities, e.g., $\exists x \Box \varphi(x)$;
- the things that we quantify vary a lot;
- the axioms depend on the special shape of φ as well;
- weak language vs. rich model: hard to axiomatize;
- fragments of FO/SO-modal language: we know little.

People involved so far

- Jie Fan, Yanjun Li, Tszyuen Lau, Shihao Xiong, Yifeng Ding, Tao Gu, Chao Xu, Xingchi Su, Jixin Liu, Zhouhang Zhou, Mo Liu, Xinyu Wang, Yu Wei, Xun Wang , Yunsong Wang, Haoyu Wang, Yiting Wang, Bo Hong...
- Hans van Ditmarsch, Malvin Gattinger, Jan van Eijck, Alexandru Baltag, Andreas Herzig, Raul Fervari, Thomas Studer, Pavel Naumov, Jia Tao, Fernando Velázquez-Quesada, Jeremy Seligman, R. Ramanujam, Padmanabh, Michael Cohen...

Some results

- **Knowing whether**: [Fan, W.& van Ditmarsch: AiML14, RSL15] [Fan & vD: ICLA15, JANCL16], [Fan 17]...
- **Knowing what**: [W. & Fan: IJCAI13, AiML14][Gu & W. AiML16], [Baltag, AiML16] [van Eijck, Gattinger, W. ICLA17]
- **Knowing how**: [W. LORI15], [W. Synthese17], [Li, W. ICLA17][Herzig, Fervari, Li, W. IJCAI17], [Fervari, Velázquez-Quesada, W. SR17][Naumov & Tao TARK17...]
- **Knowing why**: [Xu, W., Studer Synthese 19]
- **Knowing who**: [W., Seligman: AiML18]
- Special column in *Studies in Logic* by Fan, Li, Ding.

An early survey/position paper: Beyond knowing that: a new generation of epistemic logics. *Jaakko Hintikka on knowledge and game theoretical semantics*, Outstanding Contributions to Logic Series. Springer, 12: 499—533, 2018

Characteristic feature

How to distinguish the work in this line and other related work in the literature?

Whether it uses a single modality for a type of know-wh, instead of breaking it down into quantifiers, normal modalities, questions, predicates and so on.

It also gives us a new “looking glass” to understand the world.

Some knowing-wh logics we proposed and studied

wh-word	bundle	connection	key ref
whether	$\mathcal{K}\varphi \vee \mathcal{K}\neg\varphi$	non-contingency logic	[FWvD14,15
what	$\exists x\mathcal{K}(\varphi \to x \approx c)$	weakly aggregative logic	[WF13,14]
how	$\exists \pi\mathcal{K}[\pi]\varphi$	game logic, ATL	[Wang15,17
why	$\exists t\mathcal{K}(t : \varphi)$	justification logic	[XWS18]

We obtained complete axiomatizations, characterizations of expressive power, and decidability …

Along the way, we understand better why neighbourhood-like semantics works for many philosophical logic.

Example: A logic of knowing how [Fervari, Herzig, Li, W. IJCAI17]

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}p \wedge \mathcal{K}(p \to q) \to \mathcal{K}q$	NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$
T	$\mathcal{K}p \to p$	EQREPKh	$\dfrac{\varphi \to \psi}{\mathcal{K}h\varphi \to \mathcal{K}h\psi}$
4	$\mathcal{K}p \to \mathcal{K}\mathcal{K}p$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg\mathcal{K}p \to \mathcal{K}\neg\mathcal{K}p$		
AxKtoKh	$\mathcal{K}p \to \mathcal{K}hp$		
AxKhtoKKh	$\mathcal{K}hp \to \mathcal{K}\mathcal{K}hp$		
AxKhtoKhK	$\mathcal{K}\mathcal{K}hp \to \mathcal{K}h\mathcal{K}p$		
AxKhKh	$\mathcal{K}h\mathcal{K}hp \to \mathcal{K}hp$		
AxKhbot	$\neg\mathcal{K}h\bot$		

Connections to existing logics and linguistic theories

Classification by question words:

- Knowing whether: non-contingency logic, ignorance logic
- Knowing what: weakly aggregative logic, dependence logic
- Knowing how: game Logic, alternating temporal logic
- Knowing why: (quantified) justification Logic
- Knowing who: (dynamic) termed modal logic

Classification by logical forms:

- *Mention-some*: e.g., *knowing how/why*... $\exists x \mathcal{K} \varphi(x)$
- *Mention-all* (strongly exhaustive reading): e.g., I *know who* came to the party... $\forall x(\mathcal{K}\varphi(x) \vee \mathcal{K}\neg\varphi(x))$
- In-between: *know-value* $\exists x(\mathcal{K}\ c \approx x) \leftrightarrow \forall x(\mathcal{K}\ c \approx x \vee \mathcal{K}\ c \not\approx x)$

Epistemic logic: form one to many

(Routine) research questions:

- Model theory, proof theory, computational complexity
- Group knowledge
- Logical omniscience
- Natural dynamics
- Applications

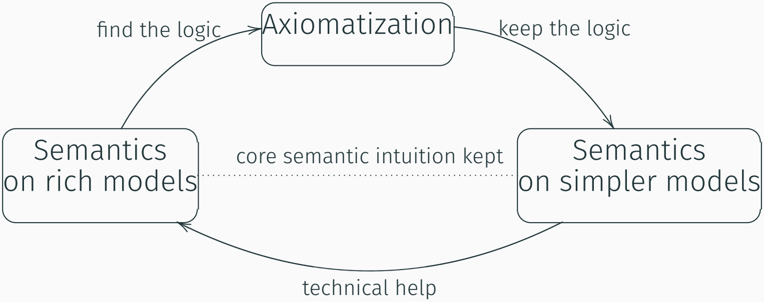
New questions:

- Interactions of different knowledge expressions;
- Simplification of semantics.

Simplify the semantics while keeping the logic

Common difficulties: weak language vs. rich semantics

To restore the balance between the language and model:



Disadvantages of those concrete logics

'Disadvantages' from a linguistic point of view:

- Compositionality
- Uniformity
- Expressivity

Disadvantages in terms of knowledge representation:

- Propositional epistemic logic is not really about the *content* of knowledge!

A question: how to explain the decidability of those logics?

Towards a general new framework

What we are after:

- Expressive enough: covering the essence of those non-standard epistemic logics
- Not too much: sharing most good properties of propositional modal logic

Uniformity, compositionality, expressvity, computability: we want a predicate modal framework like the propositional modal logic

A new framework for predicate epistemic logic

Inspired by the concrete know-wh logics, we introduce the bundle modalities into the predicate modal language:

- pack $\exists x\mathcal{K}$ into a *bundle* modality (mention-some)
- pack $\forall x\mathcal{K}w$ into a *bundle* modality (mention-all)

You can also come up with your favourite bundles inspired by the categorization of the penex forms for the *classical decision problem*.

We obtain some nice and powerful fragments of first-order modal logic.

A new framework for predicate epistemic logic

Example: epistemic language of mention-some [W. TARK17]:

$$\varphi ::= P\overline{x} \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x \mathcal{K} \varphi$$

$\exists x \mathcal{K} \varphi$: I know some thing such that φ

- "I know a theorem of which I do not know any proof": $\exists x \mathcal{K} \neg \exists y \mathcal{K} Prove(y, x)$
- "i knows a country which j knows its capital": $\exists x \mathcal{K}_i \exists y \mathcal{K}_j Capital(y, x)$

The situation for first-order modal logic is hopeless

Simply putting a decidable fragment of first-order logic plus a modality does not work at all.

Language	Decidability	Ref
P^1	undecidable	[Kripke 62]
x, y, p, P^1	undecidable	[Gabbay 93]
$x, y, \Box_i$, single P^1	undecidable	[Rybakov & Shkatov 17]

The decidable fragments are rare (only one x in $\Box$). Most of the propositional know-wh logics are in the one variable fragment.

Language	Decidability	Ref
single x	decidable	[Segerberg 73]
$x, y / P^1 /$ GF, $\Box_i(x)$	decidable	[Wolter & Zakharyaschev 01]

What about our bundled fragments?

Good news!

- $\exists\Box$ fragment is decidable over both increasing and constant domain models! $\forall x\Diamond$ weakens the power of $\forall$!
- A satisfiable $\exists\Box$ formula has a *finite tree* model.
- We have a tableau method for satisfiability of **MLMS**
- Satisfiability checking of $\exists\Box$ fragment is PSPACE-complete (exactly as the complexity of propositional model logic)
- Even you allow both $\exists\Box$ and $\forall\Box$ bundles, it is still decidable over increasing domain models.

Note that we do not need to restrict the arity of the predicates or the number of variable occurrences at all.

Bad news

The meaning of the world is the separation of wish and fact.

— Gödel

- $\exists\Box$ fragment is undecidable over S5 models.
- $\forall\Box$ fragment with two unary predicates is undecidable over constant domain models.

It is not as robust as propositional modal logic: we are at the edge of first-order expressivity.

Surprising connections

By using the epistemic bundle modalities, we discovered an intuitive way to understand a large family of non-classical logics both philosophically and mathematically:

- Intuitionistic Logic
- Inquisitive Logic
- (A version of) Truthmaker Semantics
- Possibility Semantics
- ...

The know-wh modality servers as a tool (or the missing puzzle) to crack those logics with sober technical appearances. Making the trivial things really trivial.

In the rest of the course

- Knowing whether
- Knowing what
- Knowing how
- Knowing who
- Knowing why
- Bundled fragments of FOML
- Understanding non-classical logics via know-wh