Epistemic Logic VI

Dynamic Turn (C)

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A “hybrid” example

Event models

PAL as a special ETL

System PAN

Axiom Schemas		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \chi) \to (\Box\phi \to \Box\chi)$	NECK	$\dfrac{\phi}{\Box\phi}$
DIST!	$[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$	NEC!	$\dfrac{\phi}{[\psi]\phi}$
INV	$(p \to [\psi]p) \wedge (\neg p \to [\psi]\neg p)$		
EXE	$\langle\psi\rangle\top \leftrightarrow \psi$		
NM	$\Diamond\langle\psi\rangle\phi \to [\psi]\Diamond\phi$		
PR	$\langle\psi\rangle\Diamond\phi \to \Diamond\langle\psi\rangle\phi$		

The reduction is tricky and fragile

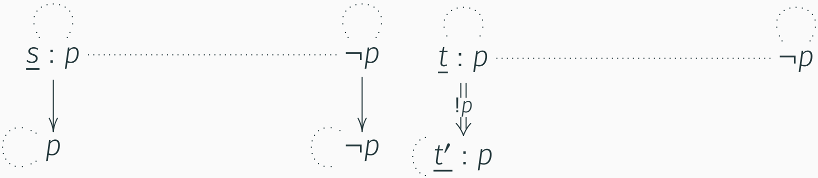
Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]\Box\phi \leftrightarrow (\psi \to \Box(\psi \to [\psi]\phi))$
Your selection	
DIST!	$[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$
NEC!	$\dfrac{\phi}{[\psi]\phi}$
RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$

A “hybrid” example

Two logical approaches about knowledge and action

	language	model	semantics
ETL	time+K	temporal+epistemic	Kripke
DEL	K+action	epistemic	Kripke+*dynamic*

$s \vDash \neg Kp \wedge F\, Kp$ $\qquad$ $t \Vdash \neg Kp \wedge [!p]Kp$



Dynamic semantics: the **meaning** of an action is the **changes** it brings to the knowledge states of the agents. (dates back to Stalnaker, Groenendijk, Stokhof and Veltman).

Halpern spent a year in Amsterdam at the birth time of DEL, but ...

A mixed-blood baby of DEL and ETL [Wang and Li, 2012]

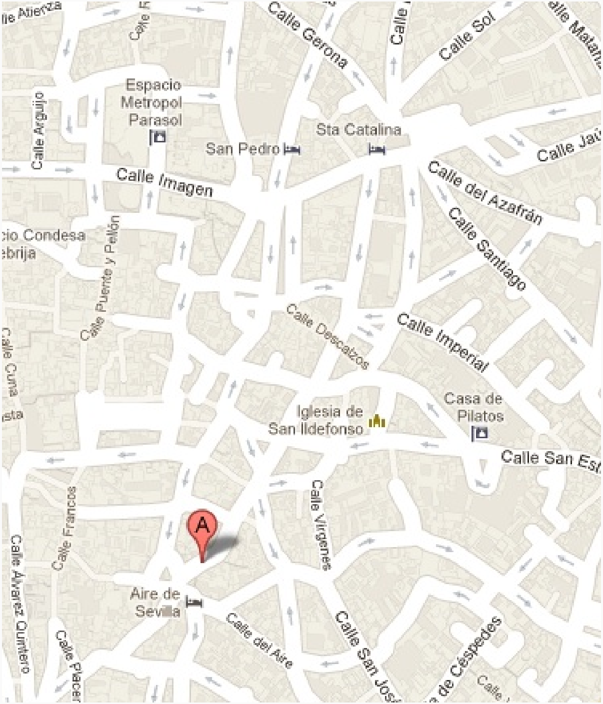
We may not construct the temporal structure from scratch.

	language	model	semantics
ETL	time+K	temporal+epistemic	Kripke
DEL	K+action	epistemic	Kripke+*dynamic*
Mixed	action+K	temporal+(initial) epistemic	Kripke+*dynamic*

Let's see an example of such a logic.

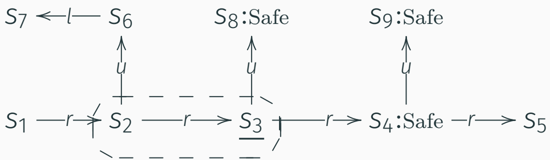

号敌楼
You Are Here
You Are Here
关王庙
You Are Here
马神庙
You Are Here
六角亭
You Are Here
迎恩坊
You Are Here
水库
水库停车场
管理处办公区
You Are Here
管理处
You Are Here
迎恩坊停车场
You Are Here
居庸关高速收费
派出所
You Are Here
@果壳趣科技
weibo.com/gizfan

Lost with a map at hand


Espacio Metropol Parasol
Calle Gerona
Calle Sol
Sta Catalina
San Pedro
Calle Arguijo
Calle Imagen
Calle del Azafrán
Calle Santiago
Calle Puente y Pellón
Calle Descalzos
Calle Imperial
Calle Cuna
Casa de Pilatos
Iglesia de San Ildefonso
Calle Francos
Calle Vírgenes
A
Aire de Sevilla
Calle Álvarez Quintero
Calle del Aire

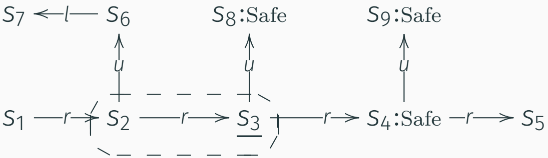
Planning under uncertainty

A rookie spy sneaking in an enemy building was guided by his headquarters. The communication with the HQ was lost at some point. Now someone spotted him and pulled the alarm. In panic he got lost...



Suppose s_3 is actually where he is, but he is not sure whether he is at s_2 or s_3 (the *bubble*).

What should he do now to be safe as quickly as possible?



Following plans can all *in fact* lead the agent to a safe place:

1. Moving right (r): the agent may not know that he is safe afterwards.
2. Moving up (u): the agent may know that he is safe *afterwards*, but he couldn't have known it beforehand
3. Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.

Plans 1 and 2 are good if the planner is the HQ. Plan 3 is good for the agent as the planner.

Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.

$$
\begin{tikzcd}
s_7 & s_6 \arrow[l, "l"'] & s_8 : \text{Safe} & s_9 : \text{Safe} & \\
s_1 \arrow[r, "r"] & s_2 \arrow[r, "r"] \arrow[u, "u"] & s_3 \arrow[r, "r"] \arrow[u, "u"] & s_4 : \text{Safe} \arrow[r, "r"] \arrow[u, "u"] & s_5
\end{tikzcd}
$$

Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.

$$
\begin{tikzcd}
s_7 & s_6 \arrow[l, "l"'] & s_8\text{:Safe} & s_9\text{:Safe} & \\
s_1 \arrow[r, "r"] & s_2 \arrow[r, "r"] \arrow[u, "u"] & s_3 \arrow[r, "r"] \arrow[u, "u"] & s_4\text{:Safe} \arrow[r, "r"] \arrow[u, "u"] & s_5
\end{tikzcd}
$$

Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.

$$
\begin{tikzcd}
S_7 & S_6 \arrow[l, "l"'] & S_8\text{:Safe} & S_9\text{:Safe} & \\
S_1 \arrow[r, "r"] & S_2 \arrow[r, "r"] \arrow[u, "u"] & S_3 \arrow[r, "r"] \arrow[u, "u"] & S_4\text{:Safe} \arrow[r, "r"] \arrow[u, "u"] & S_5
\end{tikzcd}
$$

AI planning under uncertainty

The goal of AI planning:

$$\texttt{Uncertain or false} \overset{\textit{a plan}}{\longrightarrow} \texttt{Certain and true}$$

Sources of uncertainty: initial states, observation power, non-deterministic actions

Types \ Uncertainty	Init	Obs	Act	Probability
Classical	no	full	no	no
FOND	no	full	yes	no
MDP	no	full	yes	yes
Conformant	yes	none	yes	no
Contingent	yes	partial	yes	no
POMDP	yes	partial	yes	yes

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Classical vs. conformant planning

- Classical planning: given one red to reach some blue by a sequence of actions, i.e., reachability over deterministic labelled transition systems. E.g., *rr* is a good plan.

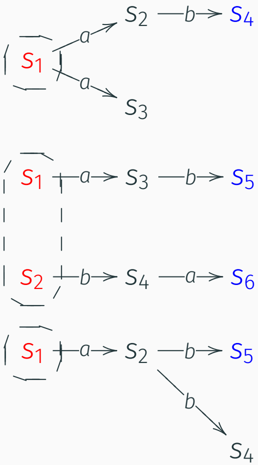
$$
\begin{tikzcd}
 & s_6 & {\color{blue}s_8} & {\color{blue}s_9} & \\
s_1 \arrow[r, "r"] & {\color{red}s_2} \arrow[r, "r"] \arrow[u, "u"] & s_3 \arrow[r, "r"] \arrow[u, "u"] & {\color{blue}s_4} \arrow[r, "r"] \arrow[u, "u"] & s_5
\end{tikzcd}
$$

- Conformant planning: given **a set** of reds find an action sequence that can **always** work no matter where to start: executable and reaching some blue when finish. E.g., *ru* is a good conformant plan but neither *u* nor *r* is good.

$$
\begin{tikzcd}
 & s_6 & {\color{blue}s_8} & {\color{blue}s_9} & \\
s_1 \arrow[r, "r"] & {\color{red}s_2} \arrow[r, "r"] \arrow[u, "u"] \arrow[ur, "u"] & {\color{red}s_3} \arrow[r, "r"] \arrow[u, "u"] & {\color{blue}s_4} \arrow[r, "r"] \arrow[u, "u"] & s_5
\end{tikzcd}
$$

More examples

Do you have a conformant plan ("tank plan") in the following models to go from red to blue?



Model: Kripke model with an uncertainty set

Given a set **P** of basic propositions and a non-empty set **A** of basic actions, an *uncertainty map (UM)*:

$$\mathcal{M} = \langle S, \{R_a \mid a \in \mathbf{A}\}, V, U \rangle$$

where $\langle S, \{R_a \mid a \in \mathbf{A}\}, V \rangle$ is a Kripke model, and $\emptyset \subset U \subseteq S$. $\mathcal{M}, s$ is a *pointed* UM model, if $s \in U$.

Example ($\mathcal{M}, s_3$)

$$\begin{tikzcd}
s_7 & s_6 \arrow[l, "l"'] & s_8{:}\mathrm{Safe} & s_9{:}\mathrm{Safe} & \\
s_1 \arrow[r, "r"] & s_2 \arrow[u, "u"] \arrow[r, "r"] & \underline{s_3} \arrow[u, "u"] \arrow[r, "r"] & s_4{:}\mathrm{Safe} \arrow[u, "u"] \arrow[r, "r"] & s_5
\end{tikzcd}$$

Epistemic Action Language

- `EAL` language with action and knowledge as modalities:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid [a]\phi \mid K\phi$$

  where $p \in \mathsf{P}$, $a \in \mathsf{A}$.

- For abbreviations: $\langle a \rangle \phi := \neg[a]\neg\phi, \hat{K}\phi := \neg K \neg \phi$.
- $K\phi$ says that the agent knows that ϕ.
- $\langle a \rangle \phi$ says that there exists an execution of a which will make ϕ true.

Semantics of ELA on UM (simplified version)

Given any UM model $\mathcal{M} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U \rangle$, the satisfaction relation on pointed UM model $\mathcal{M}, s$ is defined as:

$$\mathcal{M}, s \vDash K\phi \iff \forall u \in U : \mathcal{M}, u \vDash \phi$$

$$\mathcal{M}, s \vDash [a]\phi \iff \forall t \in S \text{ such that } s \xrightarrow{a} t, \mathcal{M}|^{a}, t \vDash \phi$$

where

- $\mathcal{M}|^{a} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U|^{a} \rangle$
- $U|^{a} = \{r' \mid \exists r \in U \text{ such that } r \xrightarrow{a} r'\}$: 'carry the bubble' further along a transitions.

You can add the observation power about the executable actions.

Examples

The truth value of EAL formulas is *not* defined on every state and it is “path dependent”:

$$\begin{tikzcd}
\underline{s_1} \arrow[r, "b"] \arrow[d, "a"] & s_3 : p \arrow[d, "a"] & \overset{b}{\Rightarrow} & s_1 \arrow[r, "b"] \arrow[d, "a"] & \underline{s_3} : p \arrow[d, "a"] \\
s_2 \arrow[ur, "a"] & s_4 & & s_2 \arrow[ur, "a"] & s_4
\end{tikzcd}$$

$$\begin{tikzcd}
\underline{s_1} \arrow[r, "b"] \arrow[d, "a"] & s_3 : p \arrow[d, "a"] & \overset{a}{\Rightarrow} \quad \overset{a}{\Rightarrow} & s_1 \arrow[r, "b"] \arrow[d, "a"] & \underline{s_3} : p \arrow[d, "a"] \\
s_2 \arrow[ur, "a"] & s_4 & & s_2 \arrow[ur, "a"] & s_4
\end{tikzcd}$$

Let the left-hand-side model be $\mathcal{M}$ then:

$\mathcal{M}|^{b}, s_3 \models Kp$ but $\mathcal{M}|^{aa}, s_3 \not\models Kp$ thus $\mathcal{M}, s_1 \models \langle b \rangle Kp \wedge \langle a \rangle \langle a \rangle \neg Kp$.

Theoretical aspects of EAL

- Normal form: K can be pushed outside $\langle a \rangle$
- A bisimualtion notion
- Finite model property
- A sound and complete axiomatization

The theoretical results can help us to understand better the logical framework that we designed and facilitate further applications.

A sound and complete axiomatization

Rules:

MP NECK NEC(a) SUB

Axioms:

TAUT	all the axioms of propositional logic
DISTK	$K(p \to q) \to (Kp \to Kq)$
NEC(a)	$[a](p \to q) \to ([a]p \to [a]q)$
T	$Kp \to p$
4	$Kp \to KKp$
5	$\neg Kp \to K\neg Kp$
PR(a)	$K[a]p \to [a]Kp$
NM(a)	$\langle a \rangle Kp \to K[a]p$

No invariance for proposition letters, no determincy, no reduction of the dynamic operator...

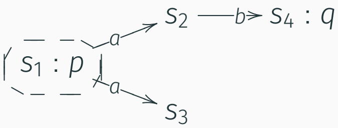
A short summary for EAL

- Model: compact (Kripke model with a bubble)
- Language: simple (K and $[a]$)
- Semantics: possible-world and dynamic
- Useful in knowledge tracking and plan verification?

Conformant planning

A (linear) *plan* is a finite sequence of actions.

Definition
Given $\mathcal{M} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U\rangle$ and a set $\emptyset \subset G \subseteq S$, find a sequence $a_1, \ldots, a_n$ such that $a_1, \ldots, a_n$ is *strongly executable* and $U|^{a_1,\ldots,a_n} \subseteq G$. Strongly executablility means for each $u \in \mathcal{U}$ $\mathcal{M}, u \vDash \llparenthesis a_1 \rrparenthesis \cdots \llparenthesis a_n \rrparenthesis \top$ where $\llparenthesis a \rrparenthesis \phi$ is the shorthand of $[a]\phi \wedge \langle a \rangle \phi$.



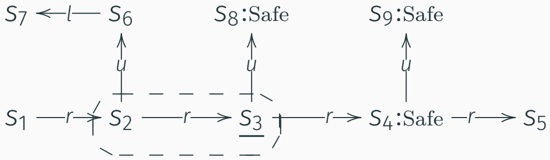
ab is not strongly executable in the above model.

In practice a *goal* is often given by a Boolean formula ϕ, and a set of actions that you can use is limited to some $\mathsf{B} \subseteq \mathsf{A}$.

Definition (Conformant planning in AI)
Given a model $\mathcal{M}$, a goal formula ϕ, and a set $\mathsf{B} \subseteq \mathsf{A}$, the conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 a_2 \cdots a_n \in \mathsf{B}^*$ such that for each $u \in \mathcal{U}_{\mathcal{M}}$ we have $\mathcal{M}, u \vDash (\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$, i.e., $\mathcal{M}, u \vDash K(\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$ for some $u \in U$. The existence problem of conformant planning is to test whether such a sequence exists.

Conformant planning

Intuitively, we want a plan which will never fail w.r.t. non-deterministic actions and initial uncertainty of the agent. E.g., *ru* is a conformant plan to the agent.



$\mathcal{M}, s_3 \vDash K(\!|r|\!)(\!|u|\!)Safe \wedge K(\!|r|\!)(\!|u|\!)KSafe$

We can **verify** conformant plans by model checking `EAL`. What about checking the existence of a plan?

To express the existence of a conformant plan

Enriched language EPDL

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid [\pi]\phi \mid K\phi$$
$$\pi ::= a \mid ?\phi \mid \pi;\pi \mid \pi \cup \pi \mid \pi^*$$

E.g., EPDL can express $K[(?\neg Kp; a)^*; ?Kp; b]K[c]q$

$\mathcal{M}, s \vDash [\pi]\phi$	$\iff$	for all $\mathcal{M}', s' : (\mathcal{M}, s)[\![\pi]\!](\mathcal{M}', s')$
		implies $\mathcal{M}', s' \vDash \phi$
$(\mathcal{M}, s)[\![a]\!](\mathcal{M}', s')$	$\iff$	$\mathcal{M}' = \mathcal{M}\vert^a$ and $s \xrightarrow{a} s'$
$(\mathcal{M}, s)[\![?\psi]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}', s') = (\mathcal{M}, s)$ and $\mathcal{M}, s \vDash \psi$
$(\mathcal{M}, s)[\![\pi_1; \pi_2]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi_1]\!] \circ [\![\pi_2]\!](\mathcal{M}', s')$
$(\mathcal{M}, s)[\![\pi_1 + \pi_2]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi_1]\!] \cup [\![\pi_2]\!](\mathcal{M}', s')$
$(\mathcal{M}, s)[\![\pi^*]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi]\!]^\star(\mathcal{M}', s')$

Express the plan existence problem

Recall: a conformant plan requires that for each $u \in \mathcal{U}_{\mathcal{M}}$ we have $\mathcal{M}, u \models (\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$.

Proposition
There exists a conformant plan w.r.t. $\mathbf{B} \subseteq \mathbf{A}$ *and a Boolean* ϕ *on* $\mathcal{M}, s$ *iff* $\mathcal{M}, s \models \langle(\Sigma_{a \in \mathbf{B}}(?K\langle a\rangle\top; a))^*\rangle K\phi$.

Call the formula $\theta_{\mathsf{B},\phi}$. If $\mathsf{B} = \{a_1, a_2\}$ then

$$\theta_{\mathsf{B},\phi} = \langle(((?K\langle a_1\rangle\top; a_1) + (?K\langle a_2\rangle\top; a_2))^*\rangle K\phi.$$

Simple-minded solution does not work

What about $K(\!|(\Sigma\mathsf{B})^*|\!)\phi$?

Example

Let $\mathcal{U} = \{s_1, s_2\}$, uncertainty map $\mathcal{M} = \langle \mathcal{N}, \mathcal{U} \rangle$ and the goal formula is p. Let $\mathsf{B} = \{a, b\}$, we have $\mathcal{M}, s_1 \vDash K(\!|\Sigma\mathsf{B}^*|\!)p$, but there is no solution to this conformant planning problem.

$$\begin{array}{ccccc} s_1 & \xrightarrow{a} & s_3 & \xrightarrow{b} & s_5 : p \\ & & & & \\ s_2 & \xrightarrow{b} & s_4 & \xrightarrow{a} & s_6 : p \end{array}$$

The last K is important

What about $\langle(\Sigma_{a\in \mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle\phi$?

Example

Let $\mathcal{U} = \{s_1\}$, and let the goal formula be p. As we can see, there is no solution to this conformant planning problem. Indeed $\mathcal{M}, s_1 \not\models \langle(\Sigma_{a\in \mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle Kp$ with $\mathsf{B} = \{a, b\}$, but we could have $\mathcal{M}, s_1 \models \langle(\Sigma_{a\in \mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle p$.

$$\begin{tikzcd}
s_1 \arrow[r, "a"] & s_2 \arrow[r, "b"] \arrow[dr, "b"] & s_5 : p \\
& & s_4
\end{tikzcd}$$

Definition (Generalized conformant planning)
Given an uncertainty map $\mathcal{M}$, a goal formula $\phi \in \texttt{EPDL}$, and a test-free (i.e., $?\phi$-free) $\pi \in \Pi_{\mathsf{A}}$, the generalized conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 \cdots a_n \in (\pi)$ such that for some $u \in \mathcal{U}_{\mathcal{M}}$, $\mathcal{M}, u \vDash K(\!|a_1|\!) \cdots (\!|a_n|\!)\phi$. The existence problem of conformant planning is to test whether such a sequence exists.

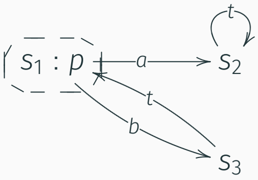
Generalizations: goal formula and plan constraint

Take p as the relief of a pain, and take q as some side effect of medicines a and b. If the goal is p then both a and b are conformant plans. If the goal is $p \land \neg Kq$, only a is a good plan.

$$
\begin{tikzcd}
s_1 \arrow[r, "a"] \arrow[dr, "b"] & s_3 : p \\
s_2 \arrow[r, "a"] \arrow[dr, "b"] & s_4 : p, q \\
& s_5 : p, q
\end{tikzcd}
$$

Generalization: goal about future (use program)

Let p express that a tooth hurts. You can either replace it with a false tooth (a) or fix the problem temporally without the replacement (b). The trouble for the second option is that it may go wrong again in some time (t). What would you choose? If your goal is $[t^*]\neg p$, which means free of worries forever, then a is clearly a better plan.



Generalization: constraints on plan

There are two kinds of transportation on the way to p: by bus (a) or by walking (b). However, you can afford taking a bus only one time. Therefore, the solution should be a sequence allowed by $\pi = b^*; a; b^* + b^*$. It is easy to see that under this constraint only $a; b$ is a plan.

$$\begin{tikzcd} s_1 \arrow[loop left] \arrow[r, "a"] & s_2 \arrow[r, bend left, "a"] \arrow[r, bend right, "b"'] & s_3 : p \end{tikzcd}$$

Generalized conformant planning as model checking

Let t be the translation of test-free programs such that each atomic action a is replaced by $(?K\langle a\rangle\top; a)$:

$$
\begin{aligned}
t(a) &= (?K\langle a\rangle\top; a)\\
t(\pi;\pi') &= t(\pi); t(\pi')\\
t(\pi+\pi') &= t(\pi) + t(\pi')\\
t(\pi^*) &= (t(\pi))^*
\end{aligned}
$$

Proposition
There exists a conformant plan w.r.t. a test-free $\pi \in \Pi_A$ and a $\phi \in \mathit{EPDL}$ on $\mathcal{M}, s$ iff $\mathcal{M}, s \vDash \langle t(\pi)\rangle K\phi$.

The standard conformant planning is under constraint $\pi = (\Sigma B)^*$.

How expensive is model checking EPDL

Theorem
Model checking $\mathtt{EPDL}$ *is* PSPACE*-complete.*

The same as standard conformant planning on labelled transition systems. Get more for free!

Usual global model checking algorithm by labelling is not applicable since the semantics of K is *path dependent*.

Advantages of such a logical approach

- much more general with the same computational price!
- natural specification of goals and constrains on plans
- specification and verification of plans with (epistemic) conditions and loops
- abstraction, refinement and equivalence of the plans
- (in principle) probability may be plugged in
- to compare complexity of different planning problems as MC of fragments over various classes of models

You have a conformant plan for ϕ $\approx$ you know how to guarantee ϕ. It inspired a logic of knowing how to be discussed later in the course.

Event models

Generalizing the public announcements

Consider the following situations:

After tossing the coin, ...

- I show it publicly to all the people
- I show it to one of the people while others are watching from distance
- I show it secretly to one of the people without others noticing
- ...

We can use the idea of Kripke models!

Events/actions are like states and you may not be sure what actually happened.

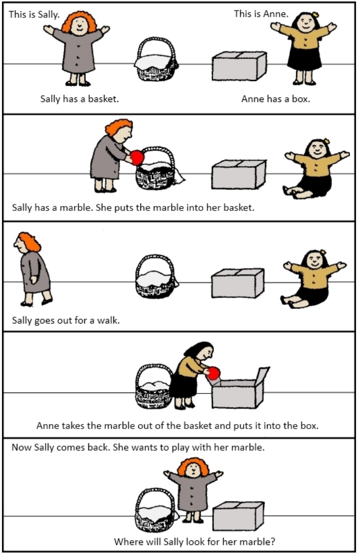
Email and social media

Managing knowledge distribution:

- Email: cc, bcc, single bulk
- WeChat: group announcements, moments
- QQ: anonymous group messages, whisper
- Zoom: Q& A list, Chat

See Yingying Cheng's master thesis.

Theory of mind



Baron-Cohen, Leslie and Frith (1985). Factual changes...

I. Dumontheil et al. (2010)
Bring the big ball up.

Event model approach [Baltag et al., 1998]

Definition (event model with factual changes)
An event model U w.r.t. language L is a triple: $\langle E, \rightarrowtail, Pre, Pos \rangle$ where:

- E is a finite non-empty set (of events);
- $\rightarrowtail : \mathsf{I} \to 2^{E \times E}$;
- $Pre : E \to \mathsf{L}$;
- $Pos : E \times \mathsf{P} \to \mathsf{L}$ assigning co-finitely many $p \in \mathsf{P}$ to itself.

$$e \overset{1}{\text{---}} e' \overset{2}{\text{---}} f$$

where $Pre(e) = Pre(e') = open$, $Pre(f) = \top$, $\mathsf{P} = \{open\}$, $Pos(e)(open) = Pos(e')(open) = \bot$ and $Pos(f)(open) = open$.

If e is the real event, then agent 1 is not sure whether agent 2 heard that someone closed the door.

Product update

In the following we assume that a language **L** and its semantics $\vDash$ is defined on Kripke models.

Definition (update product $\otimes$)
Given **P** and **I**, a Kripke model $\mathcal{M} = \langle S, \rightarrow, V\rangle$ and an event model $\mathcal{U} = \langle E, \rightarrowtail, Pre, Pos\rangle$ w.r.t. **L**, the updated Kripke model $\mathcal{M}' = \mathcal{M} \otimes E$ is a tuple $\langle S', \rightarrow', V'\rangle$ where:

- $S' = \{(s,e) \mid \mathcal{M}, s \vDash Pre(e)\}$;
- $(s,e) \rightarrow'_i (s',e')$ iff $s \rightarrow_i s'$ and $e \rightarrowtail_i e'$;
- $V((s,e))(p) = \begin{cases} 1 & \mathcal{M}, s \vDash Pos(e)(p) \\ 0 & \mathcal{M}, s \nvDash Pos(e)(p) \end{cases}$

Product $\implies$ relativization $\implies$ factual changes.

Not all the frame properties are preserved! However, it preserves bisimilarity if **L** does.

Given P and I, the *dynamic epistemic language* LDEL is defined as follows:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\mathcal{U}, e]\phi$$

where $\mathcal{U}, e$ is a pointed event model w.r.t. the language which "has been constructed", e.g., $\neg\langle\mathcal{U}, e\rangle\top$ is not a legal precondition for $\mathcal{U}$. Circular precondition can make sense sometimes.

A more precise definition (without the postconditions):

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\mathcal{F}(\phi \ldots \phi)]\phi$$

where $F \in \mathsf{F}$ denotes an event frame with n events $e_1 \ldots e_n$.

Restricting the language of pre- and postcoditions will not change the expressive power of the logic (without common knowledge)

The semantics

$$
\begin{array}{|rcl|}
\hline
\mathcal{M}, s \models \top & \Leftrightarrow & \text{always} \\
\mathcal{M}, s \models p & \Leftrightarrow & s \in V(p) \\
\mathcal{M}, s \models \neg\phi & \Leftrightarrow & \mathcal{M}, s \not\models \phi \\
\mathcal{M}, s \models \phi \wedge \psi & \Leftrightarrow & \mathcal{M}, s \models \phi \text{ and } \mathcal{M}, s \models \psi \\
\mathcal{M}, s \models \Box_i \psi & \Leftrightarrow & \forall t : s \rightarrow_i t \text{ implies } \mathcal{M}, t \models \psi \\
\mathcal{M}, s \models [\mathcal{U}, e]\phi & \Leftrightarrow & \mathcal{M}, s \models Pre(e) \text{ implies } \mathcal{M} \otimes \mathcal{U}, (s, e) \models \phi \\
\hline
\end{array}
$$

The axiom system

System $\mathbb{DEL}$

Axiom schemata		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$	NECK	$\dfrac{\phi}{\Box\phi}$
!ATOM	$[\mathcal{U}, e]p \leftrightarrow (Pre(e) \to Pos(e)(p))$	RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!NEG	$[\mathcal{U}, e]\neg\phi \leftrightarrow (Pre(e) \to \neg[\mathcal{U}, e]\phi)$		
!CON	$[\mathcal{U}, e](\phi \wedge \chi) \leftrightarrow ([\mathcal{U}, e]\phi \wedge [\mathcal{U}, e]\chi)$		
!K	$[\mathcal{U}, e]\Box\phi \leftrightarrow (Pre(e) \to \bigwedge_{f: e \rightarrowtail f} \Box[\mathcal{U}, f]\phi)$		

We can use a similar reduction to show the equivalence of expressivity between `DEL` and `EL`.

New axiomatization (given fixed $\mathcal{U}$), Wang & Auscher IJCAI 13

System **DELN**

Axiom schemata		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \chi) \to (\Box\phi \to \Box\chi)$	NECK	$\dfrac{\phi}{\Box\phi}$
DIST!	$[e](\phi \to \chi) \to ([e]\phi \to [e]\chi)$	**NEC!**	$\dfrac{\phi}{[e]\phi}$
INV	$(Pos(e)(p) \to [e]p) \land (\neg Pos(e)(p) \to [e]\neg p)$		
PRE	$\langle e\rangle\top \leftrightarrow Pre(e)$		
NM	$\Diamond\langle f\rangle\phi \to [e]\Diamond\phi$ (if $e \rightarrowtail f$ in $\mathcal{U}$)		
PR	$\langle e\rangle\Diamond\phi \to \bigvee_{f: e \rightarrowtail f} \Diamond\langle f\rangle\phi$		

PAL With common knowledge:

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \land \phi) \mid \Box_i \phi \mid \Box^*_{\mathcal{G}} \phi \mid [\phi]\phi$$

where $\mathcal{G} \subseteq \mathrm{I}$.

However, there is no reduction axiom for $[\psi]\Box^*_{\mathcal{G}}\phi$.

We need an inductive rule:

From $\chi \rightarrow [\phi]\psi$ and $\chi \land \phi \rightarrow E_{\mathcal{G}}\chi$, infer $[\psi]\Box^*_{\mathcal{G}}\phi$.

Extensions

$\mathtt{PAL}$ with relativized common knowledge (RC):

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid \Box^*_{\mathcal{G}}(\phi,\phi) \mid [\phi]\phi$$

where $\mathcal{G} \subseteq \mathsf{I}$.

$$\boxed{\begin{array}{lcl} \mathcal{M}, s \vDash \Box^*_{\mathcal{G}}(\psi,\phi) & \Leftrightarrow & \mathcal{M}, t \vDash \phi \text{ for all } t \text{ such that} \\ & & s \xrightarrow{\mathcal{G}^*} t \text{ via only } \psi \text{ worlds} \end{array}}$$

A new reduction:

$$[\psi]\Box^*_{\mathcal{G}}(\chi,\phi) \iff \Box^*_{\mathcal{G}}(\langle\psi\rangle\chi, [\psi]\phi)$$

Extensions

PDL with event models:

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid [\pi]\phi \mid [\mathcal{U}, e]\phi$$

The event model modality can be reduced by using the right updated program first (cf. [van Benthem et al., 2006]).

A new reduction:

$$[\mathcal{U}, e][\pi]\phi \leftrightarrow \bigwedge_{f \in E} [\mathtt{A}_{\mathcal{U},e,f} \times \mathtt{A}_\pi][\mathcal{U}, f]\phi$$

$\mathtt{A}_{\mathcal{U},e,f} \times \mathtt{A}_\pi$ is essentially a new program π.

Natural questions/problems

- Quantifying announcements? [Balbiani et al., 2008]
- What epistemic state can we realize by given an initial model via all kinds of events? [Bozzelli et al., 2014]
- Given an initial model will iterating an event model always stabilize? [Sadzik, 2006]
- Given an initial model and some available events, how to make sure certain epistemic goals? [Bolander and Andersen, 2011]
- What is a good notion of equivalence of event models? [van Eijck et al., 2012]
- The event models are global but not local to each agent. How to compose the global event models from locally executable ones... [van Eijck et al., 2011, French et al., 2014]

Other dynamics

See chapters in [van Benthem, 2011]

chapter	representation	dynamics
PAL	epistemic model (EM)	relativization
DEL	EM	product update
awareness	EM + accessible sets	relativization and realization
issue	EM + issue relations	link-intersection & product update
belief	EM + plausibility	lexicographic/conservative upgrade
probability	EM + probability	probabilistic product update
preference	betterness model	defined by PDL programs
games	EM + moves	relativization and product update
procedures	EM + protocols	relativization and product update
groups	doxastic model	priority update

Some further references:

`http://projects.illc.uva.nl/lgc/del/`

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