Epistemic Logic V

The Dynamic Turn (A)

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Background

Public Announcement Logic

Two basic questions to be answered

Background

Recap: Classification of logic and action

The different levels of rationality (van Benthem):

- reason logically
- act cleverly
- interact intelligently
- everything above under uncertainty

	no knowledge	knowledge	group
no action	PL	EL	...
act/time	PDL, TL	ETL, DEL, EPDL	...
strategy	ATL, STIT	AETL, ESTIT	...

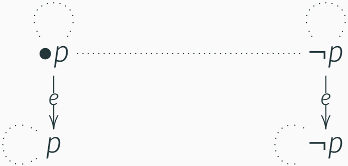
DEL stands for *Dynamic Epistemic Logic* which handles knowledge updates *constructively* and is a tool for “epistemic engineering/management” of the desired epistemic goals.

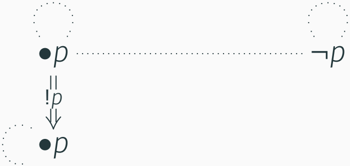
Handling knowledge changes

Epistemic Temporal Logic vs. *Dynamic* Epistemic Logic

	language	**model**	**semantics**
ETL	time+K	temporal(+epistemic)	Kripke-like
DEL	K+events	epistemic	Kripke+*dynamic*

$\neg Kp \wedge [e]\, Kp$ $\qquad\qquad$ $\neg Kp \wedge [!p]Kp$





DEL handles *how* is the knowledge updated.

A very brief pre-history

Stalnaker (1978) *Assertion*:

- Its content is *dependent* on its context.
- It *modifies* the context.

The ideas of discourse representation theory, dynamic logic and the above points together inspired the invention of *dynamic semantics* [Groenendijk and Stokhof, 1991] and *update semantics* [Veltman, 1996]:

The meaning of a sentence is identified with its *context change potential* (CCP).

(Compare it with truth conditional semantics: knowing the meaning of a sentence is knowing when it is true)

One step further:

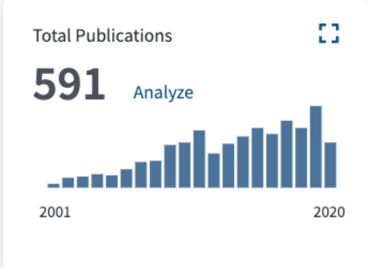
The meaning of a communicative event is the *change* it brings to the epistemic states of the participants in the discourse.

- [Gerbrandy and Groeneveld, 1997] combined the ideas of [Veltman, 1996] and [Fagin et al., 1995]: dynamic epistemic semantics for announcements.
- [Gerbrandy, 1999] developed the idea further. Some ILLC students rediscovered [Plaza, 1989] in which the public announcement logic (PAL) was proposed and studied in depth.
- [Baltag et al., 1998] proposed the dynamic epistemic logic with action model updates.

Such formal treatment of dynamics also becomes a very useful tools to understand various conditionals.

In this century

From Web of Science database:



Overview books:

- Dynamic Epistemic Logic [van Ditmarsch et al., 2007]
- Logical Dynamics of Information and Interaction [van Benthem, 2011]

Subdiscipline

From `Springer Link`

Logic and Philosophy of Language	251
Theoretical Computer Science	236
AI	206
Epistemology and Philosophy of Science	94
SWE	83
Database Management & Information Retrieval	77
Linguistics	76
Communication Networks	49
Information Systems and Applications	32
HCI	28
Game Theory	24

Let's go back to the origin...

Do we really understand thoroughly what we are doing? What is *Dynamic Epistemic Logic* as a field?

In searching for the answer, let us go back to the basics.

We will focus on axiomatizations:

- To understand the semantics-driven logics better.
- To compare with related approaches.

It is also interesting technically: dynamic semantics, failure of USUB, reduction-based axiomatization.

Public Announcement Logic

Public Announcement Logic (PAL)

The language of *Public Announcement Logic* ($\mathtt{PAL}$):

$$\phi \quad ::= \quad p \mid \neg\phi \mid (\phi \wedge \phi) \mid \mathsf{K}_i\phi \mid [\phi]\phi \text{ (also write } [!\phi]\phi)$$

We define $\langle\psi\rangle\phi$ as $\neg[\psi]\neg\phi$.

It is interpreted on ($\mathbf{S5}$) Kripke models $\mathcal{M} = (S, \{\rightarrow_i\}_{i\in I}, V)$:

$$\boxed{\begin{array}{lcl} \mathcal{M}, s \vDash \mathsf{K}_i\psi & \Leftrightarrow & \forall t : s \rightarrow_i t \implies \mathcal{M}, t \vDash \psi \\ \mathcal{M}, s \vDash [\psi]\phi & \Leftrightarrow & \mathcal{M}, s \vDash \psi \text{ implies } \mathcal{M}|_\psi, s \vDash \phi \end{array}}$$

where $\mathcal{M}|_\psi = (S', \{\rightarrow'_i \mid i \in \mathrm{I}\}, V')$ such that: $S' = \{s \mid \mathcal{M}, s \vDash \psi\}$, $\rightarrow'_i = \rightarrow_i |_{S' \times S'}$ and $V'(p) = V(p) \cap S'$.

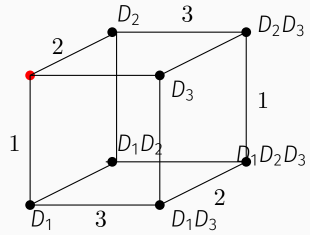
$$\begin{tikzcd} s_1 : \{p\} \arrow[loop above, "1"] \arrow[r, leftrightarrow, "1"] & s_2 : \{\} \arrow[loop above, "1"] & {[p]} \implies & s_1 : \{p\} \arrow[loop above, "1"] \end{tikzcd}$$

$$\mathcal{M}, s_1 \vDash \neg\mathsf{K}_1 p \wedge [p]\mathsf{K}_1 p$$

The classic example: Muddy Children

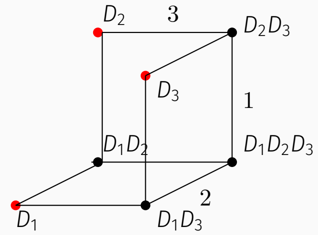
- Out of n children, $k \geq 1$ got mud on their faces while playing.
- They can see whether other kids are dirty, but there is no mirror for them to discover whether they are dirty themselves.
- Then father walks in and states: "At least one of you is dirty!" Then he requests "If you know you are dirty, step forward now."
- If nobody steps forward, he repeats his request: "If you now know you are dirty, step forward now."
- After exactly k requests to step forward, the k dirty children suddenly do so (assuming they are honest and perfect reasoners).

When there are 3 dirty children...



"At least one of you is dirty!"

Announcement: $\psi = D_1 \vee D_2 \vee D_3$



Public Announcement Logic (PAL)

The classic modal logic questions:

- Do we have a complete axiomatization?
- Do we have complete axiomatizations w.r.t. certain classes of frames ?
- Do the axioms and rules of a normal modality also hold for $[\psi]$?
- Is $\texttt{PAL}$ invariant under bisimulation or other equivalence notions?
- Does it have finite model property?
- Is it decidable?
- How is its definability over models and frames?
- What is the relationship between $\texttt{PAL}$ and modal (epistemic) logic?
- Is it translatable into first-order logic?

Get familiar with it first!

Try to get a feeling of the semantics of `PAL` by checking the validity of the following formula schemas and rules.

- $[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$, $[\psi](\phi \to \chi) \leftrightarrow ([\psi]\phi \to [\psi]\chi)$
- $\langle\phi\rangle\psi \to [\phi]\psi$, $\langle\phi\rangle\psi \to \phi$, $\langle\phi\rangle\psi \leftrightarrow (\phi \wedge [\phi]\psi)$
- $[\psi]p \leftrightarrow (\psi \to p)$, $[\psi]\neg\phi \leftrightarrow (\psi \to \neg\phi)$ ($\times$), $[\psi]\neg\phi \leftrightarrow \neg[\psi]\phi$ ($\times$), $[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
- $\dfrac{\phi}{[\psi]\phi}$, $\dfrac{\phi(p)}{\phi(\psi)}$ ($\times$), $\dfrac{\phi \leftrightarrow \psi}{[\phi]\chi \leftrightarrow [\psi]\chi}$, $\dfrac{\phi \leftrightarrow \psi}{[\chi]\phi \leftrightarrow [\chi]\psi}$
- $[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i(\psi \to [\psi]\phi))$, $[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i[\psi]\phi)$
- $[\psi][\chi]\phi \leftrightarrow [\psi \wedge \chi]\phi$ ($\times$), $[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
- $[\psi]\mathsf{K}_i\psi$ ($\times$)

Basic System PA: Axioms and Rules

Different proof systems were proposed in the literature which share the following axiom schemas and rules.

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\mathsf{K}_i(\phi \to \psi) \to (\mathsf{K}_i\phi \to \mathsf{K}_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \land \chi) \leftrightarrow ([\psi]\phi \land [\psi]\chi)$
!K	$[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i[\psi]\phi)$
Rules	
NECK	$\dfrac{\phi}{\mathsf{K}_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$

No uniform substitution!

Axioms and Rules

Axiom Schemas	
`DIST!`	$[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$
`!COM`	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
Rules	
`NEC!`	$\dfrac{\phi}{[\psi]\phi}$
`RE`	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$

Reduction /recursion axioms

Axiom Schemas	
`!ATOM`	$[\psi]p \leftrightarrow (\psi \rightarrow p)$
`!NEG`	$[\psi]\neg\phi \leftrightarrow (\psi \rightarrow \neg[\psi]\phi)$
`!CON`	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
`!K`	$[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \rightarrow \mathsf{K}_i[\psi]\phi)$

Semantic update as syntactic relativization:

$$\mathcal{M}|_{\psi}, s \vDash \phi \iff \mathcal{M}, s \vDash (\phi)^{\psi}$$

Soundness and Completeness

Proposition
All the above axiom schemas and rules are sound w.r.t the standard $\mathcal{PAL}$ *semantics.*

Theorem ([Plaza, 1989])
$\mathcal{PAL}$ *is equally expressive as basic modal logic.*

$$
\begin{array}{lcllcl}
t(p) & = & p & t([\psi]p) & = & t(\psi \to p) \\
t(\neg\phi) & = & \neg t(\phi) & t([\psi]\neg\phi) & = & t(\psi \to \neg[\psi]\phi) \\
t(\phi_1 \wedge \phi_2) & = & t(\phi_1) \wedge t(\phi_2) & t([\psi](\phi_1 \wedge \phi_2)) & = & t([\psi]\phi_1 \wedge [\psi]\phi_2) \\
t(\mathsf{K}_i\phi) & = & \mathsf{K}_i t(\phi) & t([\psi]\mathsf{K}_i\phi) & = & t(\psi \to \mathsf{K}_i[\psi]\phi) \\
 & & & t([\psi][\chi]\phi) & = & t([\psi]t([\chi]\phi))
\end{array}
$$

We can obtain another translation t' by revising t: just replace the last item by $t'([\psi][\chi]\phi) = t'([\psi \wedge [\psi]\chi]\phi)$

PAL is equally expressive as basic modal logic

Intuitively, the translation "pushes" the $[\cdot]$ modality through the formula to the inner part. How to prove that the translation will terminate and produces $[\cdot]$-free formulas?

Definition (Complexity of `PAL` formulas)

$$\begin{array}{lcl} c(\top) & = & 1 \\ c(p) & = & 1 \\ c(\neg\phi) & = & 1 + c(\phi) \\ c(\phi_1 \wedge \phi_2) & = & 1 + c(\phi_1) + c(\phi_2) \\ c(\mathsf{K}_i\phi) & = & 1 + c(\phi) \\ c([\psi]\phi) & = & (5 + c(\psi)) \cdot c(\phi) \end{array}$$

PAL is equally expressive as modal logic

We can show that:

$$
\begin{array}{lcl}
c(\phi) > c(\psi) & & \text{If } \psi \text{ is a proper subformula of } \phi \\
c([\psi]\top) & > & c(\psi \to \top) \\
c([\psi]p) & > & c(\psi \to p) \\
c([\psi]\neg\phi) & > & c(\psi \to \neg[\psi]\phi) \\
c([\psi](\phi_1 \land \phi_2)) & > & c([\psi]\phi_1 \land [\psi]\phi_2) \\
c([\psi]\mathsf{K}_i\phi) & > & c(\psi \to \mathsf{K}_i[\psi]\phi) \\
c([\psi][\chi]\phi) & > & c([\psi \land [\psi]\chi]\phi) \\
c([\psi][\chi]\phi) & > & c([\psi]t([\chi]\phi))
\end{array}
$$

PAL is equally expressive as modal logic

We can prove by induction on the **complexity** of ϕ that (cf. DEL book Lemma 7.22, 7.23):

Proposition
$t(\phi)$ and $t'(\phi)$ are $[\cdot]$-free.

We can show that:

Proposition
$\vDash \phi \leftrightarrow t(\phi)$ *and* $\vDash \phi \leftrightarrow t'(\phi)$

Is $t(\phi) = t'(\phi)$?

Recap: PA + your choice

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$K_i(\phi \to \psi) \to (K_i\phi \to K_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]K_i\phi \leftrightarrow (\psi \to K_i[\psi]\phi)$
Rules	
NECK	$\dfrac{\phi}{K_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
Your choice	
RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$

Completeness via Reduction

Completeness is proved via reduction and the completeness of basic modal logic K:

$$\vDash \phi \implies \vDash t(\phi) \overset{\text{comp. of } \mathsf{K}}{\implies} \vdash_{\mathsf{K}} t(\phi) \implies \vdash_{\mathsf{PA}^+} t(\phi) \overset{\mathit{Rd.Axioms}}{\implies} \vdash_{\mathsf{PA}^+} \phi$$

We can mimic t and t' in proof systems stronger than PA.

Proposition

$\vdash_{\mathit{PA}+\mathtt{RE}} \phi \leftrightarrow t(\phi)$ *and* $\vdash_{\mathit{PA}+\mathtt{!COM}} \phi \leftrightarrow t'(\phi)$

Theorem ([Plaza, 1989])

$\mathit{PA}+\mathtt{RE}$ is complete w.r.t. the standard semantics of $\mathtt{PAL}$.

Theorem (cf. e.g., [van Ditmarsch et al., 2007])

$\mathit{PA}+\mathtt{!COM}$ is complete w.r.t. the standard semantics of $\mathtt{PAL}$.

Public Announcement Logic (PAL)

Now we can answer most of the following questions:

- * Do we have a complete axiomatization?
- * Do we have complete axiomatizations w.r.t. other classes of frames?
- * Do the axioms and rules for K also hold for $[\psi]$?
- * Is `PAL` invariant under bisimulation?
- * Is it translatable into first-order logic?
- * Does it have finite model property?
- * Is it decidable?
- * How is its definability power (over models and frames)?

Reduction? So what?

Theorem ([Lutz, 2006])
PAL is exponentially more succinct than modal logic on arbitrary models.

$\phi_0 = \top$ and $\phi_{i+1} = \langle\langle\phi_i\rangle\Diamond_1\top\rangle\Diamond_2\top$.

Theorem ([French et al., 2011])
PAL is exponentially more succinct than modal logic on S5 models if there are more than 3 agents.

The reduction technique turns out to be extremely useful in many applications and thus dominates the field of DEL.

- Logic is more than it appears!
- Update-closeness may be considered as a desired property of a logic: it shows the logic has enough pre-encoding power [van Benthem et al., 2006].
- Compositional analysis of post-conditions.
- Difference equations?
- The orthodox programme of DEL:
  static logic+dynamic operators+reduction

Two basic questions to be answered

The first question

In some published papers, PA and its variants are mentioned as complete systems. Is PA really complete?

Unfortunately, PA and many of its “close friends” are not complete, and in some cases the flaws cannot be fixed.

The second question

Can we give meaningful axiomatizations without those reduction axioms and the reduction proof method?

Yes, we can!

We will give a general axiomatization method inspired by Epistemic Temporal Logic. It will tell us what exactly is assumed in $\mathtt{DEL}$.

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