Epistemic Logic

II. A very brief history

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Prehistory

Hintikka's foundational contributions

Early contributions by others

Prehistory

The usual disclaimer in text books on modal logic

Modal logic was discussed by several ancient authors, notably Aristotle, and also by mediaeval logicians; their work, however, lies outside the scope of this book.

—Cresswell and Hughes: *A new introduction to modal logic*

Aristotle's modal syllogism

- Chapters 3 and 8-22 of the first book of the *Prior Analytics*.
- "Aristotle's modal syllogistic is almost incomprehensible because of its many faults and inconsistencies".

  —Łukasiewicz
- Many tried to make sense of it (even in formal systems)

Natural extension of the syllogistic language:

- Quantity: all, some
- Quality: affirmative (is), negative (not)
- *Modes*: null, necessary (L), possible (M), contingent (C)
- 16 types of sentences

Aristotle's modal syllogism

Syllogisms: All A is B (A**a**B), No A is B (A**e**B), Some A is B (A**i**B), Some A is not B (A**o**B): e.g., Celarent, Darii, Ferio.

For example: B**a**rb**a**r**a** LXL is considered valid by Aristotle:

- All *B* is necessarily *A*,
- All *C* is *B*,
- therefore: All *C* is necessarily *A*.

Barbara XLL is considered invalid by Aristotle:

- All *B* is *A*,
- All *C* is necessarily *B*,
- therefore: All *C* is necessarily *A*.

De re vs. *De dicto*

- Some *A* is necessarily *B*
- Necessarily some *A* is *B*

Beyond the historical interests

In the literature on modal syllogisms, the focus has been on how to make sense of Aristotle's systems (e.g, [Malink 13]).

Inspired by the programme of *natural logic* by Larry Moss, we can take a modern view of the modal syllogisms. We made the first steps for epistemic syllogisms in [Li & Wang TARK23].

We can go much deeper beyond the original modal syllogisms by Aristotle.

Multi-agent Syllogistic with Nested Knowledge [Li & Wang 23]

Given a countable set of predicates U and a set of agents I, the language *LNES* is defined by the following grammar:

$$\varphi ::= \mathrm{All}(\mathrm{g}, \mathrm{g}) \mid \mathrm{Some}(\mathrm{g}, \mathrm{g}), \quad \mathrm{g} ::= A \mid K_i g \mid \neg g$$

Where $A \in U$ and $i \in I$.

"Something that is A is also known by i to be B": $\mathrm{Some}(\mathrm{A}, K_{\mathrm{i}}\mathrm{B})$.

"Everything i knows to be A, j also knows it": $\mathrm{All}(K_{\mathrm{i}}\mathrm{A}, K_{\mathrm{j}}\mathrm{A})$.

"Everything i knows that j knows to be A is also considered possible to be B by i": $\mathrm{All}(K_{\mathrm{i}}K_{\mathrm{j}}\mathrm{A}, \hat{K}_{\mathrm{i}}\mathrm{B})$.

The semantics is based on first-order Kripke model, as the language is essentially a fragment of first-order modal logic (with a single variable). We give sound and complete proof systems for such logics (the proofs are not easy).

ALL men by nature desire to know. An indication of this is the delight we take in our senses; for even apart from their usefulness they are loved for themselves; and above all others the sense of sight. For not only with a view to action, but even when we are not going to do anything, we prefer seeing (one might say) to everything else. The reason is that this, most of all the senses, makes us know and brings to light many differences between things.

–Aristotle *Metaphysics*

Curiosity drives the civilization further.

Later middle ages

According to Boh 1993, there are four phases of Medieval Epistemic Logic:

- Special modes: to know, to believe, to doubt, to understand...
- Knowledge of necessary propositions
- Search for necessary and sufficient conditions of knowing *contingent* propositions
- Theory of logical rules

Epistemic modes

William of Ockham:

- necessary and contingent vs. known and unknown
- modal syllogism in other modes

For example:

- Socrates is known to be a man
- Socrates is white
- therefore: Some white thing is known to be a man
- *Not* to conclude: some man is known to be white

Epistemic “axioms”

Pseudo-Scotus:

- nothing is known unless it is true
- if p and q jointly entail r then p is known and q is known jointly entail r too.
- but it does not entail r is known for:
  - p and q may be known by different people
  - $p \wedge q \rightarrow r$ is not known to the person

Epistemic “axioms”

Many authors:

- *knowing that p* is not equivalent to *knowing that p is true*.
- knowledge entails strong belief (without hesitation and fear)

Peter of Mantua:

- Strong true belief is not knowledge e.g. running Plato and Socrates
- Strong belief requires evidence for belief.

Epistemic logical rules

Strode:

- R13: From $p \to q$ infer $Kp \to Kq$
- R14: From $p \to q$ and Dq infer Dp or $K\neg p$
- R23: From $p \to q$ infer $Up \to Uq$.
- adding the epistemic subjects

Paul of Pergula: conditions of the rules (e.g., R13)

- the consequence must be sound
- the consequence must be known: $K(p \to q)$
- knowledge w.r.t. the same individual
- the subject must *understand* the proposition

Other medieval epistemic axioms in modern terms

Due to Boh 1993:

- $K_a(p \to q) \land K_a p \to K_a q$
- $K_a p \to T$ "p"
- $K_a p \to p$
- $K_a p \to K_a K_a p$
- $B_a p \to B_a B_a p$
- $K_a p \to B_a p$
- $D_a p \leftrightarrow (\neg K_a p \land \neg K_a \neg p)$
- $K_a p \to p \land B_a p \land J_a B_a p$

Recommended readings

- Ivan Boh (2000), Four Phases of Medieval Epistemic Logic. *Theoria*, 66: 129-144.
- Ivan Boh (1993): *Epistemic Logic in the Later Middle Ages*, Routledge

濠梁之辩

庄子与惠子游于濠梁之上。庄子曰：“鲦鱼出游从容，是鱼之乐也。”惠子曰：“子非鱼，安知鱼之乐？”庄子曰：“子非我，安知我不知鱼之乐？”惠子曰：“我非子，固不知子矣，子固非鱼也，子之不知鱼之乐，全矣。”庄子曰：“请循其本。子曰汝安知鱼乐云者，既已知吾知之而问我，我知之濠上也。”

知 = 箭头 + 口

Hintikka's foundational contributions

Hintikka (1962)'s foundational contributions

- motivation: logical consistency
- following von Wright's modal take on epistemic logic
- semantics for epistemic logic:
  - a single consistent set is *not* enough for epistemic possibility
  - knowing that p when $\neg p$ is not an epistemic alternative
  - alternative relations are reflexive and transitive (for knowledge, S4), serial and transitive (for belief, KD4)
- from absolute agent-less knowledge to agent-dependent knowledge
- clarify or solve philosophy problems: e.g., Moore's paradox
- discussions on issues about the rules and axioms

Philosophers were not very happy about this *semantic* approach, and rules are doubtful: e.g., logical omniscience, moreover, it is not directly relevant to the JTB approach.

Beyond “knowing that”: Hintikka’s early work

“knowing who” was discussed by Hintikka (1962) in terms of first-order modal logic: $\exists x K(Mary \approx x)$, i.e., knowing the answer of the embedded question.

Hintikka used epistemic logic to understand questions. E.g, consider the question Q : “Who murdered Mary?”

- The *presupposition* of Q is $K\exists x M(x, Mary)$.
- The *desideratum* of Q is $\exists x K M(x, Mary)$.
- One possible answer to Q is $M(John, Mary)$.
- *Conclusiveness* of the answer requires $\exists x K(John \approx x)$.
- Conclusive answers realize the desideratum ($K\exists x$ to $\exists x K$).

However, Hintikka’s work on know-wh is largely forgotten nowadays.

Know-wh inspired Hintikka to go further

- This interest in the relationship between questions and knowledge also led Hintikka to the pursuit of a *Socratic epistemology* that weighs *knowledge acquisition* more importantly than *knowledge justification*.
- The attempt to avoid quantifying higher-order entities in formalizing sentences like "I know whom every young mother should trust" leads Hintikka to the core idea of *independent friendly logic* (from $\exists f K \forall x(M(x) \to T(x, f(x)))$ to $K\forall x(\exists y/K)(M(x) \to T(x, y))$).

Jaakko Hintikka: Jan. 12, 1929 - Aug. 12, 2015

FO distributive normal form, Hintikka sets, semantics for modal logic, epistemic logic, Game semantics, interrogative model of inquiry, IF logic Math, Logic, Philosophy, Linguistics, History...

Be **Independent** and **Friendly**

Early contributions by others

Aumann (1976)'s early contributions

- motivation: information in games
- Independent from the field of logic
- state space with partitions w.r.t. different players
- no logical language: an *event* is a subset of the space
- knowledge as an operator on events:
  $K_i(E) = \{s \mid P_i(s) \subseteq E\}$
- defining a notion of common knowledge which can be computed w.r.t. finite state space.
- use the notion to show it is not possible to "agree to disagree" given common prior (probabilistic distribution of the states).

Correspondence with epistemic (S5) models.

McCarthy (1978)'s early contribution

- motivation: *who knows what* is important in AI
- puzzle collector/maker: muddy children (unfaithful wives), sum and product, the hardest logic puzzle ever
- flexibility: useful logics instead of the 'correct one'
- "any fool knows" operator O (common knowledge)

Basic axioms (1978):

- $K_a p \to p$
- $O(K_a p \to p)$
- $O(Op \to OK_a p)$
- $O(K_a p \wedge K_a(p \to q) \to K_a q)$
- optional: $O(K_a p \to K_a K_a p)$
- optional: $O(\neg K_a p \to K_a \neg K_a p)$

Puzzle: Sum and Product (wiki)

X and Y are two different natural numbers greater than 1. Their sum is not greater than 100, and Y is greater than X. S and P are two mathematicians (and consequently perfect logicians); S knows the sum $X + Y$ and P knows the product $X \times Y$. Both S and P know all the information in this paragraph.

The following conversation occurs:

- S says "P does not know X and Y."
- P says "Now I know X and Y."
- S says "Now I also know X and Y."

What are X and Y?

Work by Xiwen Ma and Weide Guo.

Early contributions by Halpern and Moses (1984, 1990)

- motivation: information in (communicating) distributed systems
- from agents to processors
- combination of time and knowledge
- dealing with logical omniscience
- constructive knowledge based on local states: interpreted systems
- study the computational aspects of epistemic logic
- show that certain coordination is not possible due to lack of common knowledge.

Conference series of TARK

- Halpern's initiation
- Bi-annal conference series on theoretical aspects of rationality and knowledge since 1986
- The 1986 conference brought computer scientists, philosopher, economists together: Aumann, Hintikka, McCarthy, Stalnaker, Halpern, Vardi, Moses, Fagin, Dwok, Smullyan, Moore, Asher, Kamp, Levesque, Immerman, Plotkin, Ladner, Fischer....
- Major interdisciplinary venue for epistemic /probabilistic reasoning in Philosophy, AI, CS, Economics, Linguistics ...
- `http://www.tark.org/`