Epistemic Logic VI

Dynamic Turn (C)

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A “hybrid” example

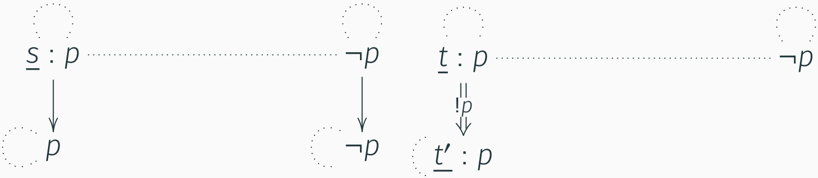
Event models

A “hybrid” example

Two logical approaches about knowledge and action

	language	model	semantics
ETL	time+K	temporal+epistemic	Kripke
DEL	K+action	epistemic	Kripke+*dynamic*

$s \vDash \neg Kp \wedge F\, Kp$ $\qquad$ $t \Vdash \neg Kp \wedge [!p]Kp$



Dynamic semantics: the **meaning** of an action is the **changes** it brings to the knowledge states of the agents. (dates back to Stalnaker, Groenendijk, Stokhof and Veltman).

Halpern spent a year in Amsterdam at the birth time of DEL, but ...

A mixed-blood baby of DEL and ETL [Wang and Li, 2012]

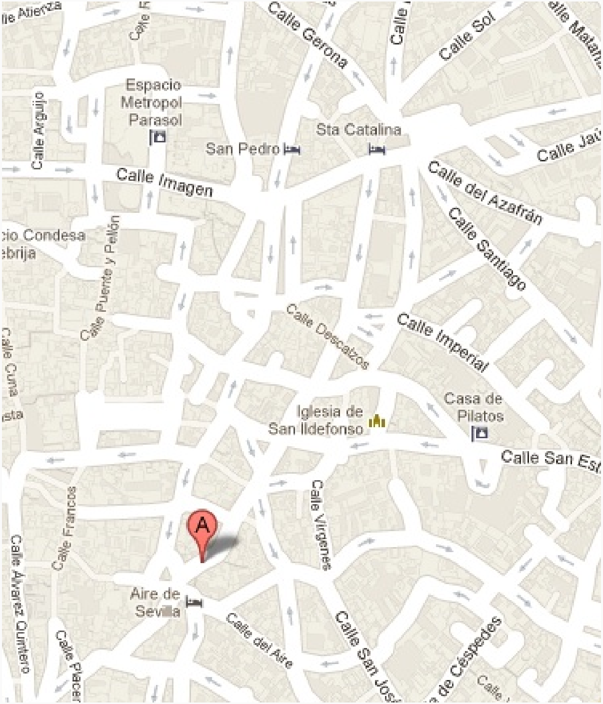
We may not construct the temporal structure from scratch.

	language	**model**	**semantics**
ETL	time+K	temporal+epistemic	Kripke
DEL	K+action	epistemic	Kripke+*dynamic*
Mixed	action+K	temporal+(initial) epistemic	Kripke+*dynamic*

Let's see an example of such a logic.

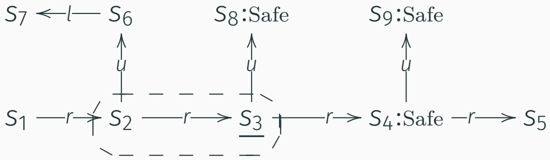
Lost with a map at hand

Lost with a map at hand


Calle Gerona
Calle Sol
Espacio
Metropol
Parasol
Calle Argüijo
Sta Catalina
San Pedro
Calle Imagen
Calle Jaú
Calle del Azafrán
Calle Santiago
Calle Puente y Pellón
Calle Descalzos
Calle Imperial
Calle Cuna
Iglesia de
San Ildefonso
Casa de
Pilatos
Calle San Est
Calle Francos
Calle Vírgenes
A
Calle Álvarez Quintero
Aire de
Sevilla
Calle del Aire
Calle San José
Calle Placer
de Céspedes

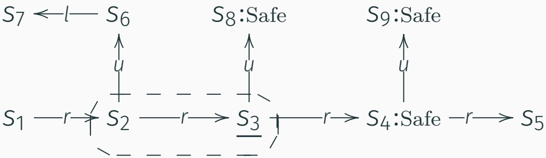
Planning under uncertainty

A rookie spy sneaking in an enemy building was guided by his headquarters. The communication with the HQ was lost at some point. Now someone spotted him and pulled the alarm. In panic he got lost...



Suppose s_3 is actually where he is, but he is not sure whether he is at s_2 or s_3 (the *bubble*).

What should he do now to be safe as quickly as possible?



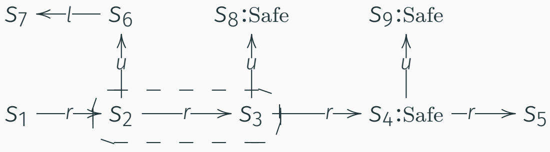
Following plans can all *in fact* lead the agent to a safe place:

1. Moving right (r): the agent may not know that he is safe afterwards.
2. Moving up (u): the agent may know that he is safe *afterwards*, but he couldn't have known it beforehand
3. Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.

Plans 1 and 2 are good if the planner is the HQ. Plan 3 is good for the agent as the planner.

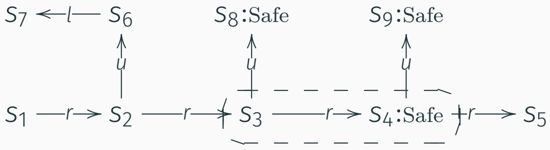
Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.



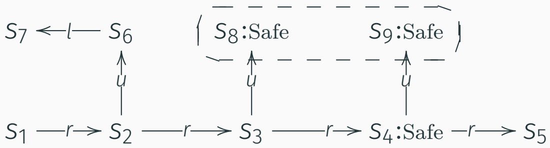
Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.



Moving r then u

Moving right and up (ru): the agent knows that it will guarantee his safety even before executing it.



AI planning under uncertainty

The goal of AI planning:

$$\texttt{Uncertain or false} \overset{\textit{a plan}}{\longrightarrow} \texttt{Certain and true}$$

Sources of uncertainty: initial states, observation power, non-deterministic actions

Types \ Uncertainty	Init	Obs	Act	Probability
Classical	no	full	no	no
FOND	no	full	yes	no
MDP	no	full	yes	yes
Conformant	yes	none	yes	no
Contingent	yes	partial	yes	no
POMDP	yes	partial	yes	yes

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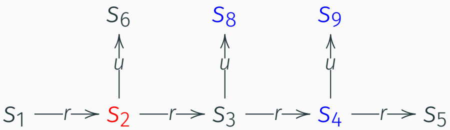
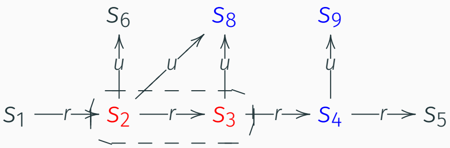
- Classical planning: given one red to reach some blue by a sequence of actions, i.e., reachability over deterministic labelled transition systems. E.g., *rr* is a good plan.

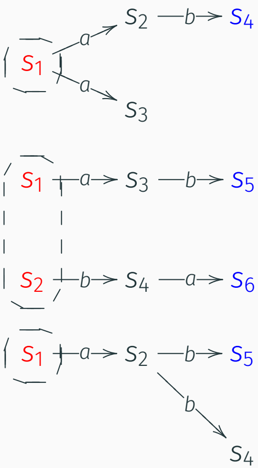


- Conformant planning: given **a set** of reds find an action sequence that can **always** work no matter where to start: executable and reaching some blue when finish. E.g., *ru* is a good conformant plan but neither *u* nor *r* is good.



More examples

Do you have a conformant plan ("tank plan") in the following models to go from red to blue?



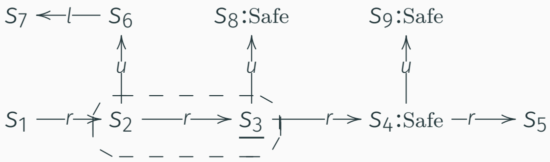
Model: Kripke model with an uncertainty set

Given a set P of basic propositions and a non-empty set A of basic actions, an *uncertainty map (UM)*:

$$\mathcal{M} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U \rangle$$

where $\langle S, \{R_a \mid a \in \mathsf{A}\}, V \rangle$ is a Kripke model, and $\emptyset \subset U \subseteq S$. $\mathcal{M}, s$ is a *pointed* UM model, if $s \in U$.

Example ($\mathcal{M}, s_3$)



Epistemic Action Language

- EAL language with action and knowledge as modalities:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid [a]\phi \mid K\phi$$

where $p \in \mathsf{P}$, $a \in \mathsf{A}$.

- For abbreviations: $\bot := \neg\top$, $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi), \phi \to \psi := \neg\phi \vee \psi, \langle a\rangle\phi := \neg[a]\neg\phi, \hat{K}\phi := \neg K\neg\phi$.
- $K\phi$ says that the agent knows that ϕ.
- $\langle a\rangle\phi$ says that there exists an execution of a which will make ϕ true.

Semantics of ELA on UM (simplified version)

Given any UM model $\mathcal{M} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U\rangle$, the satisfaction relation on pointed UM model $\mathcal{M}, s$ is defined as:

$$\mathcal{M}, s \vDash K\phi \iff \forall u \in U : \mathcal{M}, u \vDash \phi$$

$$\mathcal{M}, s \vDash [a]\phi \iff \forall t \in S \text{ such that } s \xrightarrow{a} t, \mathcal{M}|^a, t \vDash \phi$$

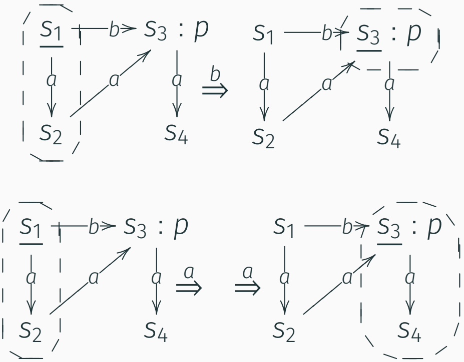
where

- $\mathcal{M}|^a = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U|^a\rangle$
- $U|^a = \{r' \mid \exists r \in U \text{ such that } r \xrightarrow{a} r'\}$: 'carry the bubble' further along a transitions.

You can add the observation power about the executable actions.

Examples

The truth value of EAL formulas is *not* defined on every state and it is “path dependent”:



Let the left-hand-side model be $\mathcal{M}$ then:

$\mathcal{M}|^b, s_3 \vDash Kp$ but $\mathcal{M}|^{aa}, s_3 \nvDash Kp$ thus $\mathcal{M}, s_1 \vDash \langle b \rangle Kp \wedge \langle a \rangle \langle a \rangle \neg Kp$.

Theoretical aspects of EAL

- Normal form: K can be pushed outside $\langle a \rangle$
- A bisimualtion notion
- Finite model property
- A sound and complete axiomatization

The theoretical results can help us to understand better the logical we designed and facilitate further applications.

A sound and complete axiomatization

Rules:

`MP` `NECK` `NEC(a)` `SUB`

Axioms:

TAUT	all the axioms of propositional logic
DISTK	$K(p \to q) \to (Kp \to Kq)$
NEC(a)	$[a](p \to q) \to ([a]p \to [a]q)$
T	$Kp \to p$
4	$Kp \to KKp$
5	$\neg Kp \to K\neg Kp$
PR(a)	$K[a]p \to [a]Kp$
NM(a)	$\langle a \rangle Kp \to K[a]p$

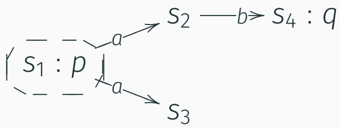
No invariance for proposition letters, no determincy, no reduction of the dynamic operator...

A short summary for EAL

- Model: compact (Kripke model with a bubble)
- Language: simple (K and $[a]$)
- Semantics: possible-world and dynamic
- Useful in knowledge tracking and plan verification?

A *plan* is a finite sequence of actions.

Definition
Given $\mathcal{M} = \langle S, \{R_a \mid a \in \mathsf{A}\}, V, U\rangle$ and a set $\emptyset \subset G \subseteq S$, find a sequence $a_1, \ldots, a_n$ such that $a_1, \ldots, a_n$ is *strongly executable* and $U|^{a_1,\ldots,a_n} \subseteq G$. Strongly executablility means for each $u \in \mathcal{U}$ $\mathcal{M}, u \vDash (\!|a_1|\!) \cdots (\!|a_n|\!)\top$ where $(\!|a|\!)\phi$ is the shorthand of $[a]\phi \wedge \langle a\rangle\phi$.



ab is not strongly executable in the above model.

Conformant planning

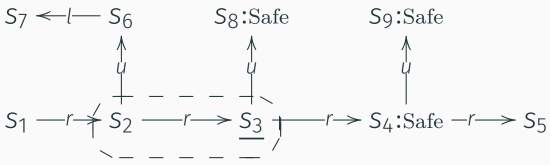
In practice a *goal* is often given by a Boolean formula ϕ, and a set of actions that you can use is limited to some $\mathsf{B} \subseteq \mathsf{A}$.

Definition (Conformant planning in AI)

Given a model $\mathcal{M}$, a goal formula ϕ, and a set $\mathsf{B} \subseteq \mathsf{A}$, the conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 a_2 \cdots a_n \in \mathsf{B}^*$ such that for each $u \in \mathcal{U}_{\mathcal{M}}$ we have $\mathcal{M}, u \vDash (\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$, i.e., $\mathcal{M}, u \vDash K(\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$ for some $u \in U$. The existence problem of conformant planning is to test whether such a sequence exists.

Conformant planning

Intuitively, we want a plan which will never fail w.r.t. non-deterministic actions and initial uncertainty of the agent. E.g., *ru* is a conformant plan to the agent.



$\mathcal{M}, s_3 \vDash K(\!|r|\!)(\!|u|\!)Safe \wedge K(\!|r|\!)(\!|u|\!)KSafe$

We can **verify** conformant plans by model checking EAL. What about checking the existence of a plan?

To express the existence of a conformant plan

Enriched language EPDL

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid [\pi]\phi \mid K\phi$$
$$\pi ::= a \mid ?\phi \mid \pi;\pi \mid \pi \cup \pi \mid \pi^*$$

E.g., EPDL can express $K[(?\neg Kp; a)^*; ?Kp; b]K[c]q$

$\mathcal{M}, s \models [\pi]\phi$	$\iff$	for all $\mathcal{M}', s' : (\mathcal{M}, s)[\![\pi]\!](\mathcal{M}', s')$
		implies $\mathcal{M}', s' \models \phi$
$(\mathcal{M}, s)[\![a]\!](\mathcal{M}', s')$	$\iff$	$\mathcal{M}' = \mathcal{M}\vert^a$ and $s \xrightarrow{a} s'$
$(\mathcal{M}, s)[\![?\psi]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}', s') = (\mathcal{M}, s)$ and $\mathcal{M}, s \models \psi$
$(\mathcal{M}, s)[\![\pi_1; \pi_2]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi_1]\!] \circ [\![\pi_2]\!](\mathcal{M}', s')$
$(\mathcal{M}, s)[\![\pi_1 + \pi_2]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi_1]\!] \cup [\![\pi_2]\!](\mathcal{M}', s')$
$(\mathcal{M}, s)[\![\pi^*]\!](\mathcal{M}', s')$	$\iff$	$(\mathcal{M}, s)[\![\pi]\!]^\star(\mathcal{M}', s')$

Express the plan existence problem

Recall: a conformant plan requires that for each $u \in \mathcal{U}_{\mathcal{M}}$ we have $\mathcal{M}, u \models (\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$.

Proposition
There exists a conformant plan w.r.t. $\mathbf{B} \subseteq \mathbf{A}$ and a Boolean ϕ on $\mathcal{M}, s$ iff $\mathcal{M}, s \models \langle(\Sigma_{a\in\mathbf{B}}(?K\langle a\rangle\top; a))^*\rangle K\phi$.

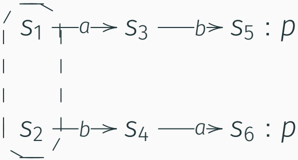
Call the formula $\theta_{\mathsf{B},\phi}$. If $\mathsf{B} = \{a_1, a_2\}$ then

$$\theta_{\mathsf{B},\phi} = \langle((?K\langle a_1\rangle\top; a_1) + (?K\langle a_2\rangle\top; a_2))^*\rangle K\phi.$$

Simple-minded solution does not work

What about $K(\!|(\Sigma\mathsf{B})^*|\!)\phi$?

Example
Let $\mathcal{U} = \{s_1, s_2\}$, uncertainty map $\mathcal{M} = \langle \mathcal{N}, \mathcal{U} \rangle$ and the goal formula is p. Let $\mathsf{B} = \{a, b\}$, we have $\mathcal{M}, s_1 \vDash K(\!|\Sigma\mathsf{B}^*|\!)p$, but there is no solution to this conformant planning problem.

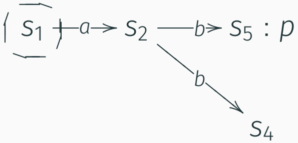


The last K is important

What about $\langle(\Sigma_{a\in\mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle\phi$?

Example

Let $\mathcal{U} = \{s_1\}$, and let the goal formula be p. As we can see, there is no solution to this conformant planning problem. Indeed $\mathcal{M}, s_1 \not\models \langle(\Sigma_{a\in\mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle Kp$ with $\mathsf{B} = \{a, b\}$, but we could have $\mathcal{M}, s_1 \models \langle(\Sigma_{a\in\mathsf{B}}(?K\langle a\rangle\top; a))^*\rangle p$.



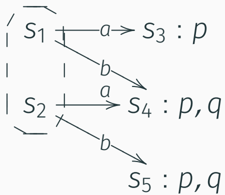
Generalized conformant planning

Definition (Generalized conformant planning)
Given an uncertainty map $\mathcal{M}$, a goal formula $\phi \in \mathsf{EPDL}$, and a test-free (i.e., $?\phi$-free) $\pi \in \Pi_{\mathsf{A}}$, the generalized conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 \cdots a_n \in (\pi)$ such that for some $u \in \mathcal{U}_{\mathcal{M}}$, $\mathcal{M}, u \vDash K(\!|a_1|\!) \cdots (\!|a_n|\!)\phi$. The existence problem of conformant planning is to test whether such a sequence exists.

Generalizations: goal formula and plan constraint

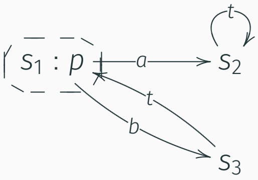
Generalization: negative epistemic goal

Take p as the relief of a pain, and take q as some side effect of medicines a and b. If the goal is p then both a and b are conformant plans. If the goal is $p \wedge \neg Kq$, only a is a good plan.



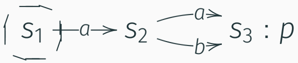
Generalization: goal about future (use program)

Let p express that a tooth hurts. You can either replace it with a false tooth (a) or fix the problem temporally without the replacement (b). The trouble for the second option is that it may go wrong again in some time (t). What would you choose? If your goal is $[t^*]\neg p$, which means free of worries forever, then a is clearly a better plan.



Generalization: constraints on plan

There are two kinds of transportation on the way to p: by bus (a) or by walking (b). However, you can afford taking a bus only one time. Therefore, the solution should be a sequence allowed by $\pi = b^*; a; b^* + b^*$. It is easy to see that under this constraint only $a; b$ is a plan.



Generalized conformant planning as model checking

Let t be the translation of test-free programs such that each atomic action a is replaced by $(?K\langle a\rangle\top; a)$:

$$t(a) = (?K\langle a\rangle\top; a)$$
$$t(\pi;\pi') = t(\pi); t(\pi')$$
$$t(\pi+\pi') = t(\pi) + t(\pi')$$
$$t(\pi^*) = (t(\pi))^*$$

Proposition

There exists a conformant plan w.r.t. a test-free $\pi \in \Pi_A$ *and a* $\phi \in \mathsf{EPDL}$ *on* $\mathcal{M}, s$ *iff* $\mathcal{M}, s \vDash \langle t(\pi)\rangle K\phi$.

The standard conformant planning is under constraint $\pi = (\Sigma \mathsf{B})^*$.

How expensive is model checking EPDL

Theorem
Model checking EPDL *is* PSPACE*-complete.*

The same as standard conformant planning on labelled transition systems. Get more for free!

Usual global model checking algorithm by labelling is not applicable since the semantics of K is *path dependent*.

Advantages of such a logical approach

- much more general with the same computational price!
- natural specification of goals and constrains on plans
- specification and verification of plans with (epistemic) conditions and loops
- abstraction, refinement and equivalence of the plans
- (in principle) probability may be plugged in
- to compare complexity of different planning problems as MC of fragments over various classes of models

You have a conformant plan for $\phi \approx$ you know how to guarantee ϕ. It inspired a logic of knowing how (Wang 15).

Event models

Generalizing the public announcements

Consider the following situations:

After tossing the coin, …

- I show it publicly to all the people
- I show it to one of the people while others are watching from distance
- I show it secretly to one of the people without others noticing
- …

We can use the idea of Kripke models!

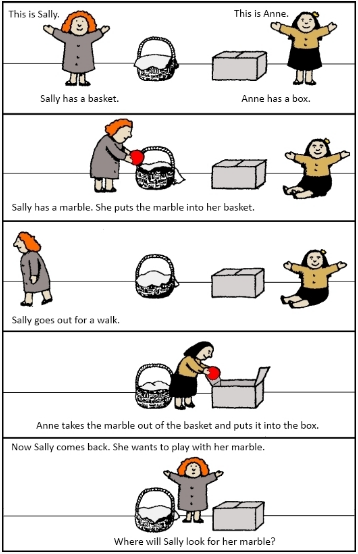
Events/actions are like states and you may not be sure what actually happened.

Email and social media

Managing knowledge distribution:

- Email: cc, bcc, single bulk
- WeChat: group announcements, moments
- QQ: anonymous group messages, whisper
- Zoom: Q& A list, Chat

See Yingying Cheng's master thesis.



I. Dumontheil et al. (2010)
Bring the big ball up.

Baron-Cohen, Leslie and Frith (1985). Factual changes...

Event model approach [Baltag et al., 1998]

Definition (event model with factual changes)
An event model U w.r.t. language L is a triple: $\langle E, \rightarrowtail, Pre, Pos\rangle$ where:

- E is a finite non-empty set (of events);
- $\rightarrowtail : \mathsf{I} \to 2^{E\times E}$;
- $Pre : E \to \mathsf{L}$;
- $Pos : E \times \mathsf{P} \to \mathsf{L}$ assigning co-finitely many $p \in \mathsf{P}$ to itself.

$$e \overset{1}{\text{——}} e' \overset{2}{\text{——}} f$$

where $Pre(e) = Pre(e') = open$, $Pre(f) = \top$, $\mathsf{P} = \{open\}$, $Pos(e)(open) = Pos(e')(open) = \bot$ and $Pos(f)(open) = open$.

If e is the real event, then agent 1 is not sure whether agent 2 heard that someone closed the door.

Product update

In the following we assume that a language **L** and its semantics $\models$ is defined on Kripke models.

Definition (update product $\otimes$)
Given **P** and **I**, a Kripke model $\mathcal{M} = \langle S, \rightarrow, V \rangle$ and an event model $\mathcal{U} = \langle E, \rightarrowtail, Pre, Pos \rangle$ w.r.t. **L**, the updated Kripke model $\mathcal{M}' = \mathcal{M} \otimes E$ is a tuple $\langle S', \rightarrow', V' \rangle$ where:

- $S' = \{(s, e) \mid \mathcal{M}, s \models Pre(e)\}$;
- $(s, e) \rightarrow'_i (s', e')$ iff $s \rightarrow_i s'$ and $e \rightarrowtail_i e'$;
- $V((s, e))(p) = \begin{cases} 1 & \mathcal{M}, s \models Pos(e)(p) \\ 0 & \mathcal{M}, s \not\models Pos(e)(p) \end{cases}$

Product $\Longrightarrow$ relativization $\Longrightarrow$ factual changes.

Not all the frame properties are preserved! However, it preserves bisimilarity if **L** does.

Dynamic Epistemic Logic with event models

Given P and I, the *dynamic epistemic language* LDEL is defined as follows:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\mathcal{U}, e]\phi$$

where $\mathcal{U}, e$ is a pointed event model w.r.t. the language which "has been constructed", e.g., $\neg\langle\mathcal{U}, e\rangle\top$ is not a legal precondition for $\mathcal{U}$. Circular precondition can make sense sometimes.

A more precise definition (without the postconditions):

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\mathcal{F}(\phi \ldots \phi)]\phi$$

where $F \in$ F denotes an event frame with n events $e_1 \ldots e_n$.

Restricting the language of pre- and postcoditions will not change the expressive power of the logic (without common knowledge)

The semantics

$$
\begin{array}{rcl}
\mathcal{M}, s \models \top & \Leftrightarrow & \text{always} \\
\mathcal{M}, s \models p & \Leftrightarrow & s \in V(p) \\
\mathcal{M}, s \models \neg\phi & \Leftrightarrow & \mathcal{M}, s \not\models \phi \\
\mathcal{M}, s \models \phi \wedge \psi & \Leftrightarrow & \mathcal{M}, s \models \phi \text{ and } \mathcal{M}, s \models \psi \\
\mathcal{M}, s \models \Box_i \psi & \Leftrightarrow & \forall t : s \rightarrow_i t \text{ implies } \mathcal{M}, t \models \psi \\
\mathcal{M}, s \models [\mathcal{U}, e]\phi & \Leftrightarrow & \mathcal{M}, s \models Pre(e) \text{ implies } \mathcal{M} \otimes \mathcal{U}, (s, e) \models \phi
\end{array}
$$

The axiom system

System $\mathbb{DEL}$

Axiom schemata		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$	NECK	$\dfrac{\phi}{\Box\phi}$
!ATOM	$[\mathcal{U}, e]p \leftrightarrow (Pre(e) \to Pos(e)(p))$	RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!NEG	$[\mathcal{U}, e]\neg\phi \leftrightarrow (Pre(e) \to \neg[\mathcal{U}, e]\phi)$		
!CON	$[\mathcal{U}, e](\phi \wedge \chi) \leftrightarrow ([\mathcal{U}, e]\phi \wedge [\mathcal{U}, e]\chi)$		
!K	$[\mathcal{U}, e]\Box\phi \leftrightarrow (Pre(e) \to \bigwedge_{f: e \rightarrowtail f} \Box[\mathcal{U}, f]\phi)$		

We can use a similar reduction to show the equivalence of expressivity between DEL and EL.

New axiomatization (given fixed $\mathcal{U}$), Wang & Aucher IJCAI 13

System **DELN**

Axiom schemata		Rules	
TAUT	all the instances of tautologies	MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
DISTK	$\Box(\phi \to \chi) \to (\Box\phi \to \Box\chi)$	NECK	$\dfrac{\phi}{\Box\phi}$
DIST!	$[e](\phi \to \chi) \to ([e]\phi \to [e]\chi)$	**NEC!**	$\dfrac{\phi}{[e]\phi}$
INV	$(Pos(e)(p) \to [e]p) \land (\neg Pos(e)(p) \to [e]\neg p)$		
PRE	$\langle e \rangle \top \leftrightarrow Pre(e)$		
NM	$\Diamond\langle f \rangle\phi \to [e]\Diamond\phi$ (if $e \rightarrowtail f$ in $\mathcal{U}$)		
PR	$\langle e \rangle\Diamond\phi \to \bigvee_{f: e \rightarrowtail f} \Diamond\langle f \rangle\phi$		

Extensions

PAL With common knowledge:

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid \Box^*_{\mathcal{G}}\phi \mid [\phi]\phi$$

where $\mathcal{G} \subseteq$ I.

However, there is no reduction axiom for $[\psi]\Box^*_{\mathcal{G}}\phi$.

We need an inductive rule:

From $\chi \to [\phi]\psi$ and $\chi \wedge \phi \to E_{\mathcal{G}}\chi$, infer $[\psi]\Box^*_{\mathcal{G}}\phi$.

Extensions

PAL with relativized common knowledge (RC):

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid \Box^*_{\mathcal{G}}(\phi, \phi) \mid [\phi]\phi$$

where $\mathcal{G} \subseteq \mathsf{I}$.

$$\boxed{\begin{array}{lcl} \mathcal{M}, s \vDash \Box^*_{\mathcal{G}}(\psi, \phi) & \Leftrightarrow & \mathcal{M}, t \vDash \phi \text{ for all } t \text{ such that} \\ & & s \xrightarrow{\mathcal{G}^*} t \text{ via only } \psi \text{ worlds} \end{array}}$$

A new reduction:

$$[\psi]\Box^*_{\mathcal{G}}(\chi, \phi) \iff \Box^*_{\mathcal{G}}(\langle\psi\rangle\chi, [\psi]\phi)$$

Extensions

PDL with event models:

$$\phi ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid [\pi]\phi \mid [\mathcal{U}, e]\phi$$

The event model modality can be reduced by using the right updated program first (cf. [van Benthem et al., 2006]).

A new reduction:

$$[\mathcal{U}, e][\pi]\phi \leftrightarrow \bigwedge_{f \in E} [\mathsf{A}_{\mathcal{U},e,f} \times \mathsf{A}_{\pi}][\mathcal{U}, f]\phi$$

$\mathsf{A}_{\mathcal{U},e,f} \times \mathsf{A}_{\pi}$ is essentially a new program π.

Natural questions/problems

- Quantifying announcements? [Balbiani et al., 2008]
- What epistemic state can we realize by given an initial model via all kinds of events? [Bozzelli et al., 2014]
- Given an initial model will iterating an event model always stabilize? [Sadzik, 2006]
- Given an initial model and some available events, how to make sure certain epistemic goals? [Bolander and Andersen, 2011]
- What is a good notion of equivalence of event models? [van Eijck et al., 2012]
- The event models are global but not local to each agent. How to compose the global event models from locally executable ones... [van Eijck et al., 2011, French et al., 2014]

Other dynamics

See chapters in [van Benthem, 2011]

chapter	representation	dynamics
PAL	epistemic model (EM)	relativization
DEL	EM	product update
awareness	EM + accessible sets	relativization and realization
issue	EM + issue relations	link-intersection & product update
belief	EM + plausibility	lexicographic/conservative upgrade
probability	EM + probability	probabilistic product update
preference	betterness model	defined by PDL programs
games	EM + moves	relativization and product update
procedures	EM + protocols	relativization and product update
groups	doxastic model	priority update

Some further references:

`http://projects.illc.uva.nl/lgc/del/`

Balbiani, P., Baltag, A., van Ditmarsch, H., Herzig, A., Hoshi, T., and de Lima, T. (2008).
'knowable' as 'known after an announcement'.
Rev. Symb. Log., 1(3):305–334.

Baltag, A., Moss, L., and Solecki, S. (1998).
The logic of public announcements, common knowledge, and private suspicions.
In *Proceedings of TARK '98*, pages 43–56. Morgan Kaufmann Publishers Inc.

Bolander, T. and Andersen, M. B. (2011).
Epistemic planning for single and multi-agent systems.
J. Appl. Non Class. Logics, 21(1):9–34.

Bozzelli, L., van Ditmarsch, H., French, T., Hales, J., and Pinchinat, S. (2014).

Refinement modal logic.
Inf. Comput., 239:303–339.

French, T., Hales, J., and Tay, E. (2014).
A composable language for action models.
In Goré, R., Kooi, B. P., and Kurucz, A., editors, *Advances in Modal Logic 10*, pages 197–216. College Publications.

Sadzik, T. (2006).
Exploring the iterated update universe.

van Benthem, J. (2011).
Logical dynamics of information and interaction.
Cambridge University Press.

van Benthem, J., van Eijck, J., and Kooi, B. (2006).
Logics of communication and change.
Information and Computation, 204(11):1620–1662.

van Eijck, J., Ruan, J., and Sadzik, T. (2012).
Action emulation.
Synth., 185(Supplement-1):131–151.

van Eijck, J., Sietsma, F., and Wang, Y. (2011).
Composing models.
J. Appl. Non Class. Logics, 21(3-4):397–425.

Wang, Y. and Li, Y. (2012).
Not all those who wander are lost: Dynamic epistemic reasoning in navigation.
In *Advances in Modal Logic 9*, pages 559–580. College Publications.