Advanced Modal Logic

Conclusions, Reflections, and Further Pointers

Yanjing Wang
Department of Philosophy, Peking University
wangyanjing.com

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(Not So) Advanced Modal Logic

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What is modal logic?

Important Ideas

Further readings

Reflections

What is modal logic?

- Modal logic is non-classical.
- Modal logic is the logic of "necessity" and "possibility".
- Modal logic is obtained from propositional logic by adding a modality operator $\Box$.
- Modal logic is a fragment of first-order logic/MSO.
- Modal logic is useful in many application areas in CS.
- Modal logic is in PSPACE.

Features in each component of $(\mathsf{L}, \mathbb{C}, \vDash, \vdash)$. It has many good properties: decidability, finite tree model property, compactness, interpolation, neat characterization ...

Modal Logic as an umbrella term for logic studies related to modalities (and possible worlds).

(Basic normal) Modal logic is:

- The bisimulation invariant fragment of FOL over models
- The maximal compact logic invariant under bisimulation
- Disguised Boolean Algebra with Operators

Important Ideas

Important proof ideas:

曲线救国 (detour), 从小看老 (construct ultrafilter), 里应外合 (closure), 一石激起千层浪 (satisfiable theory), 游戏人生 (games), 左右互搏 (Bisimulation), 深入浅出 (strengthen the hypothesis), 以点代面 (Jankov-Fine), 以偏概全 (Sahlqvist, incompleteness), 破镜重圆 (disjoint union of generated submodels), 画地为牢 (definability results), 警报范围 (Otto's proof), 循序渐进 (Lindenbaum)，天天向上 (step by step), 改头换面 + 添砖贴瓦 (transforming cannonical models), 铺陈展开 (Unraveling bulldozing), 追根溯源 (Kracht)，形意变换 ((in)completnesss), 潜在公式 (ultrafilter extension), 几乎所有 (ultraproduct), 大姐替二姐 (Sahlqvist diamond), 框架换模型 (Goldblatt), 表达变区分 (van Benthem), 函数改关系 (bisimulation), 以小见大 (filtration), 以大见小 (validity preservation), 一言以蔽之 (characterizing formula), 树比格子好 (decidability/complexity), 时空转换 (PSPACE), 余是补的归纳 (coinduction), 具体表抽象 (algebraic logic), 言无尽而意有穷 (finite many formulas/types modulo

## Some conceptual points (beyond modal logic)

- See the same thing from different perspectives.
- To really understand an apple, you probably need a pear.
- A notion of fruits help you to see more ($L, C, \vDash, \vdash$).
- Design a looking glass to focus on the things you care.
- Intuitive semantics matters.
- More constraints push you to understand deeper.
- Recognize the importance of ideas in definitions, theorem statements, and connections between things.
- Define things based on how you plan to use them.

Some conceptual points

- Negative results also demonstrate importance of logic.
- Often infinite objects are better behaved than finite ones.
- Syntax vs. Semantics, $\exists$ vs. $\forall$, finite vs. infinite.
- Use a logical framework: model checking, automated reasoning, conceptual modelling...
- The driving force of new developments of logic often comes from CS (they usually don't care what is logic).
- Philosophical logic opens a door to rigorous conceptual analysis.
- Jump out of the box and ask *why not*.
- Have an open mind but don't drop the brain....

- Help philosophers formalize their theories.
- Provide computer scientists tools to make rather philosophical concept implementable.
- Leave new interesting mathematical questions open.
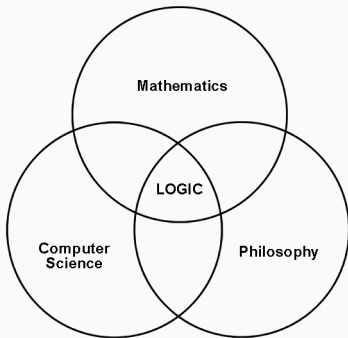
Further readings

What we did not cover

- Non-classical logics
  - Intuitionistic logic: embedding and Kripke semantics
  - Conditional logics
- Other semantics
  - Coalgebraic modal logic
  - Provability logic
- Extensions
  - Hybrid logic
  - Modal Mu-calculus
  - Multi-dimensional modal logic
  - …

What we did not cover

- Proof theory of modal logic
- Meta structure of logics
- First-order modal logic
- ML of specific domains, e.g., temporal logic (CS), epistemic logic, deontic logic...
- Applications: description logic, formal semantics

📄 Johan van Benthem
Modal Logic for Open Minds. CSLI, 2010.

📄 Patrick Blackburn, Johan van Benthem, Frank Wolter (eds.)
Handbook for Modal Logic . Elsevier, 2006.

📄 D. M. Gabbay, F. Guenthner (eds.)
Handbook of Philosophical Logic (2nd edition) Springer

📄 M.J. Cresswell, G.E. Hughes
A New Introduction to Modal Logic. Routledge, 1996

📄 Melvin Fitting and Richard L. Mendelsohn
First-Order Modal Logic, Kluwer Academic Publishers, 1998.

📄 Brian Chellas
Modal Logic: An Introduction. Cambridge University Press, 1980.

📄 Marcus Kracht
Tools and Techniques in Modal Logic, Elesvier, 1999.

📄 Alexander Chagrov, Michael Zakharyaschev
Modal Logic, Clarendon Press, 1997.

📄 A. Kurucz F. Wolter M. Zakharyaschev Dov M. Gabbay
Many-Dimensional Modal Logics: Theory and Applications
Elesvier, 2003

📄 James Garson
Modal Logic for Philosophers Cambridge University Press, 2006.

📄 Timothy Williamson
Modal Logic as Metaphysics. Oxford University Press, 2013

To know the recent developments

The delay in academia...

- Look at papers in *Advances in Modal Logic*
  (http://www.aiml.net)
- Subscribe to the google scholar alert with key words such as "modal logic"
- Others conferences and journals:
  - Conferences: LICS, CSL, MFCS, IJCAI, AAAI, KR, DL, AAMAS, TARK, LOFT, LORI...
  - Journals: Review of Symbolic Logic, Synthese, Studia Logica、Annals of Pure and Applied Logic, Journal of Philosophical Logic, Notre Dame Journal of Formal Logic, ACM Transactions on Computational Logic, Journal of Logic and Computation, Artificial Intelligence...

Reflections

- 知识储备：知道经典模态逻辑的基本结果和重要定理的证明策略，以及一些关键定义的发现过程。知道"两条线"。
- 逻辑技能：对于新的模态语言知道可以问什么技术问题，能解决一些问题，对问题的难易程度及关键点有正确的评估，形成一定的"数学直观"。有基础技术才能用起来。
- 学术技能：熟练阅读英文教材及文献，能用 LaTeX 写作业做 slides（可用 overleaf 入门），能独立做学术报告，知道如何搜索、下载文献及相关资料。
- 态度培养：不畏惧复杂问题，认真细致，虚心但不盲从，有一个开放心态。愿意用逻辑工具形式化地分析问题，学以致用。不一定特别快，但要尽可能深刻，不怕犯错，但要争取不断进步。逐渐形成自己的学术"品位"。

- Online vs. Offline
- P vs. NP
- Textbooks vs. research papers
- Covering all vs. being selective
- Amount of homework

Let me know your opinion.

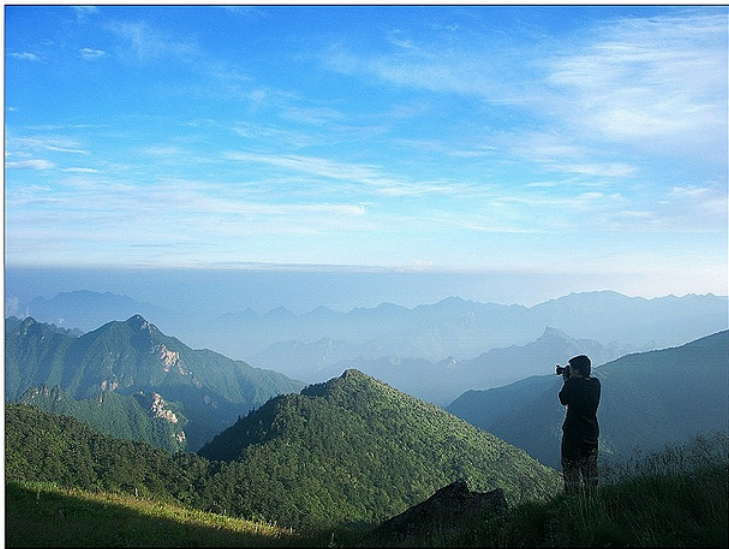Next semester: **Epistemic logic** (Wed. 15:10-18:00)

- You can use your knowledge of modal logic to study a formal account of knowledge (in a multi-agent setting).
- and its application in Philosophy, TCS, AI, Game theory
- Beyond the propositional modal logic:
    - Knowing what/how/why/what/who...
    - Decidable fragments of First-order modal logic
- A concrete way to understand many existing modal logics.
- Understand better the world around you.
- Make your own contributions!

Hopefully we can go back to classrooms....

Acknowledgements

- TA: Xun Wang
- You guys: for your patience and hard work!

Modal logic is still a very young field with lots of opportunities (especially first-order modal logic and polyadic modal logic)

ZXCV at Greenlife