Logical Dynamics IV: an application

Dynamic epistemic reasoning in navigation

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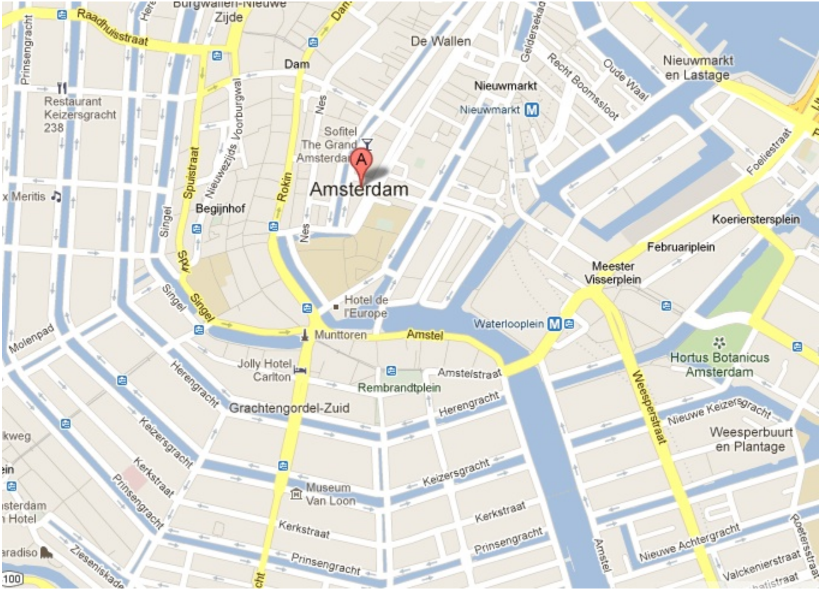
Based on the joint work with Yanjun Li:

Not all those who wander are lost: dynamic epistemic reasoning in navigation. *Advances in Modal Logic* Vol.9: 559–580.

Lost with a map at hand

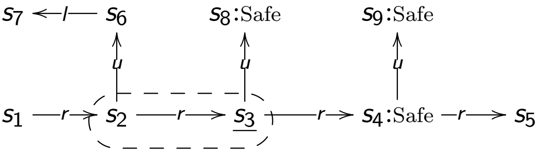
Lost with a map at hand


Raadhuisstraat
Zijde
Dam
De Wallen
Nieuwmarkt
Nieuwmarkt
Nieuwmarkt en Lastage
Recht Boomssloot
Oude Waal
Prinsengracht
Restaurant Keizersgracht 238
Nes
Sofitel The Grand Amsterdam
Amsterdam
Nieuwezijds Voorburgwal
Spuistraat
Rokin
Begijnhof
Singel
Nes
Koerierstersplein
Februariplein
Meester Visserplein
Foeliestraat
Hotel de l'Europe
Waterlooplein
Singel
Singel
Molenpad
Munttoren
Amstel
Hortus Botanicus Amsterdam
Jolly Hotel Carlton
Amstelstraat
Rembrandtplein
Herengracht
Grachtengordel-Zuid
Herengracht
Weesperstraat
Nieuwe Keizersgracht
Weesperbuurt en Plantage
Keizersgracht
Kerkstraat
Prinsengracht
Museum Van Loon
Keizersgracht
Kerkstraat
Kerkstraat
Prinsengracht
Prinsengracht
Amstel
Nieuwe Achtergracht
Valckenierstraat
Roetersstraat
Zieseniskade

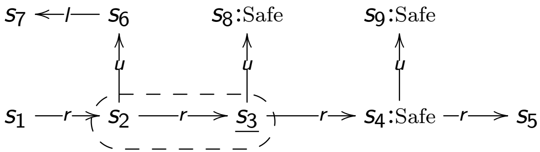

Calle Gerona
Calle Sol
Espacio Metropol Parasol
Calle Arguijo
Sta Catalina
San Pedro
Calle Imagen
Calle del Azafrán
Calle Santiago
Calle Puente y Pellón
Calle Descalzos
Calle Imperial
Calle Cuna
Casa de Pilatos
Iglesia de San Ildefonso
Calle Francos
Calle Virgenes
A
Aire de Sevilla

Scenario in *Mission Impossible*

A secret agent sneaking in an enemy building is guided by his headquarters. Usually the communication with the HQ will be lost at some point, then the agent needs to find his own way.



Scenario in *Mission Impossible*



- The agent may move right, but then he may not know that he is safe.
- The agent may move up, and then he should know that he is safe.
- Agent may plan to move right then up to guarantee his safety.

Our motivation and goal

A little bit of reasoning may help us to accomplish the mission.

- Motivation: formalize the reasoning patterns in such scenarios.
- Long term goal: apply the formalization to navigation and planning problems in real life, when GPS is not available.

Related work

- Dynamic Epistemic logic (DEL) ([Pla89, BM04, vDvdHK07])
- Epistemic Temporal logic (ETL) proposed in [FHMV95, PR85]
- Merging the two [vBGHP09, Hos09]
- Algebraic approaches inspired by DEL [PPPS10, PS10, Hor11]
- Belief space based conformant planning e.g., [BG00]

Our semantic-driven approach is a careful blend of DEL and ETL

- temporal information in the models
- dynamic semantics
- can be automated
- with simple model and natural language

Model: Kripke model with an uncertainty set

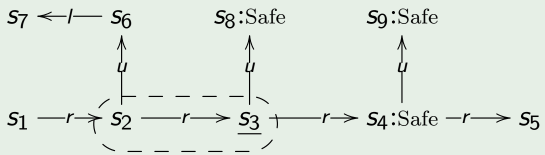
Given a set **P** of basic propositions and a *finite* set **A** of basic actions:

- A (non-deterministic) transition system: $\mathcal{N} = \langle S, \{R_a \mid a \in \mathbf{A}\}, V\rangle$ where $S \neq \emptyset$, $R_a \subseteq S \times S$ and $V : \mathbf{P} \to \mathcal{P}(S)$.
- An *uncertainty map (UM)*:

$$\mathcal{M} = \langle S, \{R_a \mid a \in \mathbf{A}\}, V, U\rangle$$

  where $U \neq \emptyset$, $U \subseteq S$ such that $\forall s, t \in S$, $e(s) = e(t)$.
- $\mathcal{M}, s$ is a *pointed* UM model, if $s \in U$.

Example ($\mathcal{M}, s_3$)



Epistemic Action Language

- EAL language with action and knowledge as modalities:

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi \mid K\phi$$

  where $p \in \mathbf{P}$, $a \in \mathbf{A}$.

- For abbreviations: $\bot := \neg\top$, $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi), \phi \to \psi := \neg\phi \vee \psi, [a]\phi := \neg\langle a \rangle \neg\phi$, $\hat{K}\phi := \neg K \neg\phi$.

- The intuition of the formula:
  - $K\phi$ says that the agent knows that ϕ
  - $\langle a \rangle \phi$ says that it is possible that after doing a, ϕ holds.

Semantics of EAL on UM

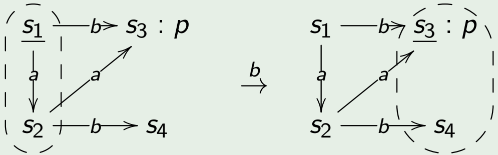
Given any UM model $\mathcal{M} = \langle S, \{R_a \mid a \in \mathbf{A}\}, V, U\rangle$, the satisfaction relation on pointed UM model $\mathcal{M}, s$ is defined as:

$$\mathcal{M}, s \vDash K\phi \iff \forall u \in U : \mathcal{M}, u \vDash \phi$$

$$\mathcal{M}, s \vDash \langle a\rangle\phi \iff \exists t \in S \text{ such that } s \xrightarrow{a} t \text{ and } \mathcal{M}|^a_t, t \vDash \phi$$

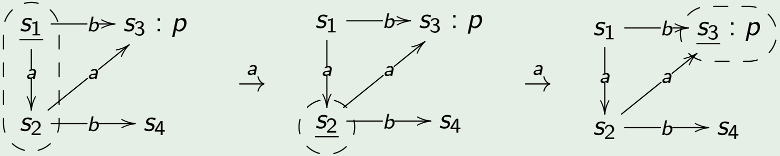
- $\mathcal{M}|^a_t = \langle S, \{R_a \mid a \in \mathbf{A}\}, V, U|^a_t\rangle$
- $U|^a_t = U|^a \cap E(t)$ 'carry' the circle along $\xrightarrow{a}$, and then check with the observations at the current location
- $U|^a = \{r' \mid \exists r \in U \text{ such that } r \xrightarrow{a} r'\}$
- $E(t) = \{t' \mid e(t') = e(t)\}$

Example



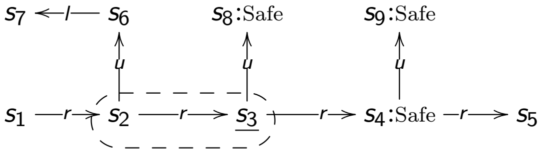
$\mathcal{M}, s_1 \vDash K\neg p \wedge \langle b\rangle\neg Kp$ and $\mathcal{M}|^b_{s_3}, s_3 \vDash \neg Kp$

Example (path dependency)



$\mathcal{M}, s_1 \vDash K\neg p \wedge \langle a\rangle\langle a\rangle Kp$ and $(\mathcal{M}|^a_{s_2})|^a_{s_3}, s_3 \vDash Kp$

The scenario in *Mission Impossible*



- $\mathcal{M}, s_3 \vDash \langle r \rangle(\text{Safe} \wedge \neg K\text{Safe})$
  (HQ guides you safe but you do not know it)
- $\mathcal{M}, s_3 \vDash \langle u \rangle(\text{Safe} \wedge K\text{Safe}) \wedge \neg K[u]\text{Safe}$
  (HQ guides you safe and you know it)
- $\mathcal{M}, s_3 \vDash K([r][u]K\text{Safe} \wedge \langle r \rangle\langle u \rangle K\text{Safe})$
  (You know the plan will make you safe)

We can use our framework to *verify* whether a plan can guarantee a goal via model checking, but it is not that trivial. We will come back to it later.

A sound and complete axiomatization $\mathbf{S}_{\mathtt{EAL}}$

Axioms:

TAUT	axioms of propositional logic
DISTK	$K(p \to q) \to (Kp \to Kq)$
DIST(a)	$[a](p \to q) \to ([a]p \to [a]q)$
T	$Kp \to p$
4	$Kp \to KKp$
5	$\neg Kp \to K\neg Kp$
OBS(a)	$K\langle a\rangle\top \vee K\neg\langle a\rangle\top$
PR(a)	$\langle a\rangle\hat{K}p \to \hat{K}\langle a\rangle p$
NM(a)	$\bigwedge_{\mathbf{B}\subseteq\mathbf{A}}(\hat{K}\langle a\rangle(p \wedge \psi_{\mathbf{B}}) \to [a](\psi_{\mathbf{B}} \to \hat{K}p))$

Rules:

MP NECK NEC(a) SUB

where $\psi_{\mathbf{B}}$ is $\bigwedge_{a\in\mathbf{B}}\langle a\rangle\top \wedge \bigwedge_{a\notin\mathbf{B}}\neg\langle a\rangle\top$

Completeness

We cannot prove the completeness by a usual reduction argument as in the standard DEL. We will prove it directly by constructing a model for each maximal consistent set of formulas.

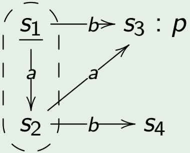
An auxiliary semantics of EAL on *epistemic multi-modal models (EM models)*.

- Structure: $\mathcal{N}_{EM} = \langle S, \{R_a \mid a \in \mathbf{A}\}, V, \sim \rangle$ where $\sim$ is an equivalent relation such that $s \sim t$, implies $e(s) = e(t)$.
- The satisfaction relation $\Vdash$:
  - $\mathcal{N}, s \Vdash \langle a \rangle \phi \iff \exists t : s \xrightarrow{a} t$ and $\mathcal{N}, t \Vdash \phi$
  - $\mathcal{N}, s \Vdash K\phi \iff \forall t : s \sim t$ implies $\mathcal{N}, t \Vdash \phi$

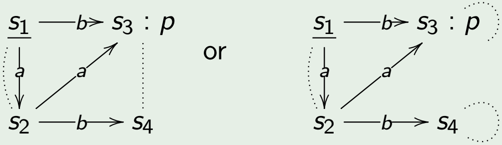
Actually, $\Vdash$ is the standard semantics for normal modal logic.

Example

In general, we cannot 'saturate' an UM model into a logically equivalent EM model. E.g., consider the UM model $\mathcal{N}$:



and we have that $\mathcal{N}, s_1 \vDash \langle b \rangle \neg Kp \wedge \langle a \rangle \langle a \rangle Kp$.
We cannot saturate $\mathcal{N}$ with some more equivalence classes such that $\mathcal{N}' \Vdash \langle b \rangle \neg Kp \wedge \langle a \rangle \langle a \rangle Kp$.



However, we can show that the two semantics do coincide on the canonical EM model.

Proposition (using $\texttt{PR}(a)$ and $\texttt{NL}(a)$)

In the canonical (EM) model $\mathcal{C}$, if $s \xrightarrow{a} t$, then $(U_s)|^a \cap E(t) = U_t$. Namely $U_s|^a_t = U_t$ and thus $\mathcal{C}_s|^a_t = \mathcal{C}_t$.

Lemma

For any $\mathcal{EAL}$ formula ϕ, any s in $\mathcal{C}$: $\mathcal{C}, s \Vdash \phi \iff \phi \in s$.

Lemma

For any $\mathcal{EAL}$ formula ϕ, any s in $\mathcal{C}$: $\mathcal{C}, s \Vdash \phi \iff \mathcal{C}_s, s \vDash \phi$

$$\begin{array}{ccc} \mathcal{C}, s \Vdash \phi & \Longleftrightarrow & \phi \in s \\ \Updownarrow & \swarrow\!\!\nearrow & \\ \mathcal{C}_s, s \vDash \phi & & \end{array}$$

Structure invariance

Given an UM model $\mathcal{M} = \langle S, \{R_a\}_{a \in \mathbf{A}}, U, V \rangle$, let $\mathcal{M}^{\mathrm{ML}} = \langle S, \{R_a\}_{a \in \mathbf{A}}, V \rangle$.

Definition

For any $\mathcal{M} = \langle S, \{R_a\}_{a \in \mathbf{A}}, U, V \rangle$, $\mathcal{M}' = \langle S', \{R'_a\}_{a \in \mathbf{A}}, U', V' \rangle$, we say that $\mathcal{M}$ is U-bisimilar to $\mathcal{M}'$ (notation: $\mathcal{M} \rightleftarrows \mathcal{N}$) iff:

- for any $u \in U$, there is a $u' \in U'$, such that $\mathcal{M}^{\mathrm{ML}}, u \leftrightarroweq \mathcal{N}^{\mathrm{ML}}, u'$,
- for any $u' \in U'$, there is a $u \in U$, such that $\mathcal{M}^{\mathrm{ML}}, u \leftrightarroweq \mathcal{N}^{\mathrm{ML}}, u'$.

We say two pointed UM models are *U-bisimilar* ($\mathcal{M}, u \rightleftarrows \mathcal{N}, u'$) iff $\mathcal{M}^{\mathrm{ML}}, u \leftrightarroweq \mathcal{N}^{\mathrm{ML}}, u'$ and $\mathcal{M} \rightleftarrows \mathcal{N}$.

Proposition

For any pointed UM models $\mathcal{M}, u, \mathcal{N}, u'$: $\mathcal{M}, u \rightleftarrows \mathcal{N}, u'$ implies $\mathcal{M}, u \equiv_{EAL} \mathcal{N}, u'$. If the models are image-finite then the converse holds.

Normal form

The above proposition of structure invariance says that the distinguishing power of EAL is the same as its fragment where knowledge operator only appears outside the action modalities. Formally, formulas ϕ in this fragment (EALK) can be generated by:

$$\begin{array}{lcl} \phi & ::= & \top \mid p \mid \psi \mid \neg\phi \mid \phi \wedge \phi \mid K\phi \\ \psi & ::= & \top \mid p \mid \neg\psi \mid \psi \wedge \psi \mid [a]\psi \end{array}$$

Next we will show that every EAL formula is equivalent to an (exponentially longer) EALK formula, namely they have the same expressive power.

Theorem

For any EAL formula ϕ, there is an (exponentially longer) EALK formula ϕ', such that $\vDash \phi \leftrightarrow \phi'$.

Finite model property

A proof base on the proposition of normal form.

- $\vDash$ and $\Vdash$ coincide on $\mathtt{EALK}$ formulas.
- For any $\mathtt{EALK}$ formula ϕ, ϕ has a UM model iff ϕ has an EM model (w.r.t. $\Vdash$)
- $\mathtt{EALK}$ on EM model has the finite model property (an easy exercise for normal modal logic)
- Any pointed EM model of an $\mathtt{EALK}$-formula can be viewed as an $\mathtt{EALK}$-equivalent UM model by ignoring the equivalence classes that do not contain the designated point.

Comparison with ETL

Technically speaking, a single-agent ETL model is just a tree-like EM model. We can unravel a UM model into such an ETL model.

Definition

Given a UM model $\mathcal{M} = \langle S, \{R_a \mid a \in \mathbf{A}\}, U, V \rangle$, we define $\mathcal{M}^{\mathrm{ETL}}$ as $\langle S^\bullet, \{R^\bullet_a \mid a \in \mathbf{A}\}, \sim, V^\bullet \rangle$ where:

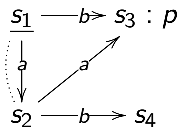
1. $S^\bullet = \{\rho \mid \rho$ is a path in $\mathcal{M}$ starting with some $s \in U\}$
2. $\rho \xrightarrow{a} \rho'$ in $\mathcal{M}^{\mathrm{ETL}}$ iff $\rho' = \rho a t$ for some $t \in S$ and $a \in \mathbf{A}$.
3. For any two paths $\rho = s_0 a_1 \cdots a_n s_n$ and $\rho' = t_0 b_1 \cdots b_m t_m$ in $S^\bullet$: $\rho \sim \rho'$ in $\mathcal{M}^\bullet$ iff ($n = m$, and for all $i \leq n$: $a_i = b_i$ and $e(s_i) = e(t_i)$).
4. $V^\bullet(s_0 a_1 \cdots a_n s_n) = V(s_n)$

Theorem

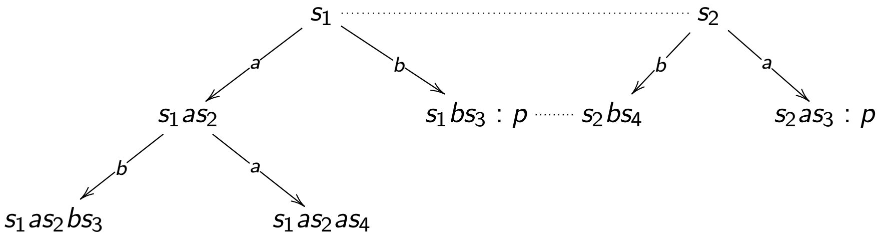
For any EAL formula ϕ: $\mathcal{M}, s \vDash \phi \iff \mathcal{M}^\bullet, s \Vdash \phi$.

An example

An UM model: $\mathcal{M}, s_1 \vDash K\neg p \wedge \langle b \rangle \neg Kp$



The ETL model: $\mathcal{M}^{\mathrm{ETL}}, s_1 \Vdash K\neg p \wedge \langle b \rangle \neg Kp$



A short summary

- Logic EAL
  - Model: intuitive and compact
  - Language: natural and succinct
  - Semantics: dynamic and epistemic
- Properties of EAL
  - Axiamatization
  - Structural invariance
  - Normal form
  - Finite model property
- Comparisons with ETL and DEL

Our framework is a careful blend of ETL and DEL enjoying the good features from the both.

Conformant planning: based on ongoing work with Quan Yu and Yanjun Li.

Let $(\!|a|\!)\phi$ be the shorthand of $[a]\phi \wedge \langle a \rangle \phi$.

Definition (Conformant planning)

Given an uncertainty map $\mathcal{M}$, a goal formula ϕ, and a set $\mathbf{B} \subseteq \mathbf{A}$, the conformant planning problem is to find a finite (possibly empty) sequence $\sigma = a_1 a_2 \cdots a_n \in \mathcal{L}(\mathbf{B}^*)$ such that for each $u \in \mathcal{U}_{\mathcal{M}}$ we have $\mathcal{M}, u \vDash (\!|a_1|\!)(\!|a_2|\!) \cdots (\!|a_n|\!)\phi$. The existence problem of conformant planning is to test whether such a sequence exists.

Intuitively, we want a plan which is both executable and safe w.r.t. non-deterministic actions and initial uncertainty of the agent.

How to check there exists such a plan?

Epistemic PDL

Enriched language EPDL

$$\phi ::= \top \mid p \mid \neg\phi \mid \phi \wedge \phi \mid [\pi]\phi \mid K\phi$$
$$\pi ::= a \mid ?\phi \mid \pi;\pi \mid \pi \cup \pi \mid \pi^*$$

$$\begin{array}{rcl}
\mathcal{M}, s \models [\pi]\phi & \iff & \text{for all } \mathcal{M}', s' : (\mathcal{M}, s)[\![\pi]\!](\mathcal{M}', s') \\
& & \text{implies } \mathcal{M}', s' \models \phi \\
(\mathcal{M}, s)[\![a]\!](\mathcal{M}', s') & \iff & \mathcal{M}' = \mathcal{M}|^a \text{ and } s \xrightarrow{a} s' \\
(\mathcal{M}, s)[\![?\psi]\!](\mathcal{M}', s') & \iff & (\mathcal{M}', s') = (\mathcal{M}, s) \text{ and } \mathcal{M}, s \models \psi \\
(\mathcal{M}, s)[\![\pi_1;\pi_2]\!](\mathcal{M}', s') & \iff & (\mathcal{M}, s)[\![\pi_1]\!] \circ [\![\pi_2]\!](\mathcal{M}', s') \\
(\mathcal{M}, s)[\![\pi_1 + \pi_2]\!](\mathcal{M}', s') & \iff & (\mathcal{M}, s)[\![\pi_1]\!] \cup [\![\pi_2]\!](\mathcal{M}', s') \\
(\mathcal{M}, s)[\![\pi^*]\!](\mathcal{M}', s') & \iff & (\mathcal{M}, s)[\![\pi]\!]^\star(\mathcal{M}', s')
\end{array}$$

without observing the available actions...

The right formula to check for the existence of a conformant plan w.r.t. $\mathbf{B} \subseteq \mathbf{A}$ and $\phi \in \mathtt{EPDL}$ is:

$$\theta_{\mathbf{B},\phi} = \langle (\Sigma_{a \in \mathbf{B}}(?K\langle a \rangle \top; a))^{*} \rangle K\phi.$$

For example, if $\mathbf{B} = \{a_1, a_2\}$ then
$\theta_{\mathbf{B},\phi} = \langle (((?K\langle a_1 \rangle \top; a_1) + (?K\langle a_2 \rangle \top; a_2))^{*} \rangle K\phi$.

Intuitively, the conformant plan consists of actions that are always executable given the uncertainty of the agent (guaranteed by the guard $K\langle a \rangle \top$). In the end the plan should also make sure that ϕ must hold given the uncertainty of the agent (guaranteed by $K\phi$). How hard is model checking EPDL on uncertainty maps?

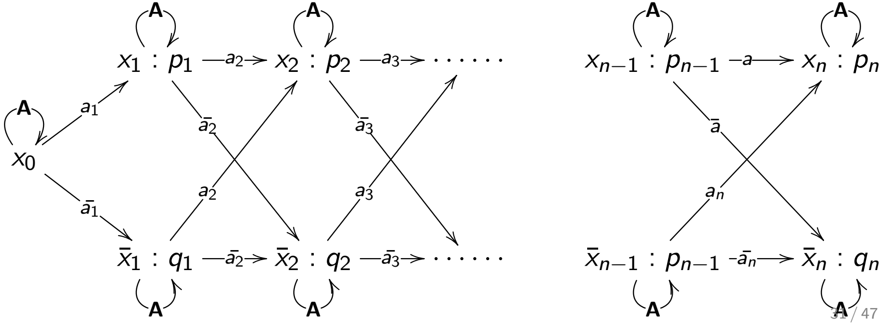
PSPACE lower bound

Given a formula α in the shape $Q_1 x_1 Q_2 x_2 \dots Q_n x_n \phi(x_1, \dots, x_n)$ where: Q_i is $\exists$ if i is odd and Q_i is $\forall$ if i is even and ϕ is a CNF based on variables $x_1, \dots, x_n$.

The validity checking of α can be reduced to model checking

$$\langle (a_1 + \bar{a}_1); ?(p_1 \vee q_1) \rangle [(a_2 + \bar{a}_2); ?(p_2 \vee q_2)] \cdots \psi(p_1, \cdots, p_n, q_1, \cdots, q_n)$$

where: $\psi(p_1, \cdots, p_n, q_1, \cdots, q_n)$ is obtained from $\phi(x_1, \dots, x_n)$ by replacing each x_i with $\hat{K} p_i$ and replacing each $\neg x_i$ with $\hat{K} q_i$.



Advantages of a logical approach:

- various ways to do planning: model checking, SAT checking, theorem proving (well studied in TCS)
- complicated goals: not just a set of states
- specification of plans with (epistemic) conditions and loops
- abstraction, refinement and equivalence of the plans
- in principle, probability can be plugged in
- to compare complexity of different planning problems as MC of fragments over various sub classes of models.

Future work

- Complexity of satisfiability
- Axiomatization of EPDL
- Contingent planning
- Probabilistic planning
- Multi-agent
- Incomplete map
- Orientation

All That Is Gold Does Not Glitter by J. R. R. Tolkien

All that is gold does not glitter, 金子未必都发光,
Not all those who wander are lost; 游民未必是流氓。
The old that is strong does not wither, 老当益壮葆青春,
Deep roots are not reached by the frost. 根深蒂固经风霜。
From the ashes a fire shall be woken, 死灰复燃火势旺,
A light from the shadows shall spring, 昏天暗地光清扬。
Renewed shall be blade that was broken, 宝剑锋处断錾出,
The crownless again shall be king. 无冕之王又做庄!

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