

Logical Dynamics III: axiomatizations of PAL

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- 1 Axiomatizations via reduction
- 2 A new axiomatization

Recap: Public Announcement Logic (PAL)

The language of *Public Announcement Logic (PAL)*:

$$\phi ::= \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\phi]\phi \text{ (also write } [!\phi]\phi)$$

It is interpreted on **(S5)** Kripke models $\mathcal{M} = (S, \{\rightarrow_i\}_{i \in I}, V)$:

$$\boxed{\begin{array}{l} \mathcal{M}, s \models \Box_i\psi \Leftrightarrow \forall t : s \rightarrow_i t \implies \mathcal{M}, t \models \psi \\ \mathcal{M}, s \models [\psi]\phi \Leftrightarrow \mathcal{M}, s \models \psi \text{ implies } \mathcal{M}|_\psi, s \models \phi \end{array}}$$

where $\mathcal{M}|_\psi = (S', \{\rightarrow'_i \mid i \in I\}, V')$ such that: $S' = \{s \mid \mathcal{M}, s \models \psi\}$, $\rightarrow'_i = \rightarrow_i \upharpoonright_{S' \times S'}$ and $V'(p) = V(p) \cap S'$.

$$\begin{array}{ccc} \begin{array}{c} \curvearrowright \\ s_1 : \{p\} \end{array} & \leftarrow 1 \rightarrow & \begin{array}{c} \curvearrowright \\ s_2 : \{\} \end{array} & [p] \implies & \begin{array}{c} \curvearrowright \\ s_1 : \{p\} \end{array} \end{array}$$

$$\mathcal{M}, s_1 \models \neg\Box_1 p \wedge [p]\Box_1 p$$

Recap: **PA** + your choice

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \rightarrow p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \rightarrow \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]K_i\phi \leftrightarrow (\psi \rightarrow K_i(\psi \rightarrow [\psi]\phi))$
Rules	
NECK	$\frac{\phi}{K_i\phi}$
MP	$\frac{\phi, \phi \rightarrow \psi}{\psi}$
Your choice	
RE	$\frac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[x/\phi]}$
!COM	$[\psi][x]\phi \leftrightarrow [\psi \wedge [\psi]x]\phi$

First question

Now what about the core system **PA**? Is it complete?

In some published papers and books, **PA** is also mentioned as a complete system for PAL. Many people use the **PA**-like systems to axiomatize other dynamic epistemic logics.

Unfortunately, **PA** and many of its “close friends” are not complete,

First question

Is **PA** complete?

$$\vDash \phi \implies \vDash t(\phi) \xrightarrow{\text{completeness of } \mathbf{K}} \vdash_{\mathbf{K}} t(\phi) \implies \vdash? t(\phi) \xrightarrow{\text{Rd.Axioms}} \vdash? \phi$$

$$t([\psi][x]\phi) = t([\psi]t([x]\phi))$$

$$t'([\psi][x]\phi) = t'([\psi \wedge [\psi]x]\phi)$$

The first translation needs RE, the second translation needs !COM in the proof system.

Negative Answer

PA is not complete!

We need to show that there exists ϕ : $\vDash \phi$ but $\not\vdash_{\mathbf{PA}} \phi$.

A general proof strategy to show some formula is not derivable in a proof system **S**: design a non-standard semantics \Vdash which validates the axioms and rules in **S**.

Thus for all ϕ : $\vdash_{\mathbf{S}} \phi \implies \Vdash \phi$. Then from $\not\vdash \phi$ we have $\not\vdash_{\mathbf{S}} \phi$.

A non-standard semantics

Goal: design a semantics to validate **PA** but not !COM (nor RE).
 Given a Kripke model over $\mathcal{M} = (S, \{\rightarrow_i \mid i \in I\}, V)$, the truth value of a PAL formula ϕ at a state s in \mathcal{M} is recursively defined as based on \Vdash_ρ where ρ is a formula in the language of PAL:

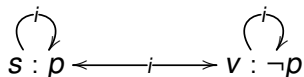
$\mathcal{M}, s \Vdash \phi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_{\top} \phi$
$\mathcal{M}, s \Vdash_\rho \top$	\Leftrightarrow	always
$\mathcal{M}, s \Vdash_\rho \rho$	\Leftrightarrow	$\rho \in V(s)$
$\mathcal{M}, s \Vdash_\rho \neg\phi$	\Leftrightarrow	$\mathcal{M}, s \not\Vdash_\rho \phi$
$\mathcal{M}, s \Vdash_\rho \phi \wedge \psi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_\rho \phi$ and $\mathcal{M}, s \Vdash_\rho \psi$
$\mathcal{M}, s \Vdash_\rho [\psi]\phi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_{\top} \psi$ implies $\mathcal{M}, s \Vdash_{\rho \wedge \psi} \phi$
$\mathcal{M}, s \Vdash_\rho \Box_i \phi$	\Leftrightarrow	$\forall t \triangleright_i s : \mathcal{M}, t \Vdash_{\top} \rho$ implies $\mathcal{M}, t \Vdash_\rho \phi$

We say ϕ is *valid* w.r.t. \Vdash if $\Vdash \phi$ (equivalently $\Vdash_{\top} \phi$).

It is handy to show $\Vdash \rho \leftrightarrow \rho'$ then $\mathcal{M}, s \Vdash_\rho \phi \Leftrightarrow \mathcal{M}, s \Vdash_{\rho'} \phi$ for any ϕ and any \mathcal{M}, s .

Some examples

Consider the following (S5) model \mathcal{M} with two worlds s, v :



$\mathcal{M}, s \Vdash \neg \Box_i p \iff \mathcal{M}, s \not\Vdash_{\top} \Box_i p \iff (\exists t \triangleright_i s : \mathcal{M}, t \Vdash_{\top} \top$
 $\text{and } \mathcal{M}, t \not\Vdash_{\top} p)$. Since $p \notin V(v)$ and $s \xrightarrow{i} v$, $\mathcal{M}, s \Vdash \neg \Box_i p$.

$\mathcal{M}, s \Vdash_p \Box_i p \iff (\forall t \triangleright_i s : \mathcal{M}, t \Vdash_{\top} p \text{ implies } \mathcal{M}, t \Vdash_p p)$.

Clearly, $\mathcal{M}, s \Vdash_p \Box_i p$. Similarly $\mathcal{M}, s \Vdash_{\top \wedge p} \Box_i p$.

$\mathcal{M}, s \Vdash [p][\neg \Box_i p] \perp \iff (\mathcal{M}, s \Vdash_{\top} p \text{ implies } \mathcal{M}, s \Vdash_{\top \wedge p} [\neg \Box_i p] \perp) \iff$
 $(\mathcal{M}, s \Vdash_{\top} \neg \Box_i p \text{ implies } \mathcal{M}, s \Vdash_{\top \wedge p \wedge \neg \Box_i p} \perp)$. Thus $\mathcal{M}, s \not\Vdash [p][\neg \Box_i p] \perp$.

On the other hand, it is easy to verify that

$\mathcal{M}, s \Vdash [p \wedge [p] \neg \Box_i p] \perp$.

$$!COM : [\psi][x]\phi \leftrightarrow [\psi \wedge [\psi]x]\phi \quad \times$$

$$!COM\wedge : [\psi][x]\phi \leftrightarrow [\psi \wedge x]\phi$$

Theorem

For all PAL formulas ϕ : $\vdash_{\mathbf{PA}+DIST!} \phi$ implies $\Vdash \phi$.

Lemma

None of $!COM$, $NEC!$, $RE!$, RE is valid under \Vdash .

Theorem

$\mathbf{PA} + DIST!$ is not complete.

Theorem

$\mathbf{PA} + !COM\wedge$ is sound and complete w.r.t. \Vdash .

To complete the whole picture

We have seen that **PA** + **DIST!** is not complete but **PA** + **NEC!** + **DIST!** is complete (why?). So, what about **PA** + **NEC!**? We need to design a new semantics.

Theorem

***DIST!** is not derivable from **PA** + **NEC!**.*

As an immediate corollary:

Corollary

***PA** + **NEC!** is not complete w.r.t. standard semantics \models .*

Conclusion of the answer to question 1

Summary of the results (**PA** can be replaced by **PAS5**, see [Wang and Cao, 2013]):

derivable/admissible in PA	not derivable/admissible in PA
WDIST!, FUNC, RE \neg , RE \wedge , RE \square	!COM, DIST!, SDIST!, PRE, !K', NEC!, RE!, RE
sound & complete systems	sound & incomplete systems
PA -!CON+DIST!+NEC!, PA +PRE+NEC! PA +RE, PA +!COM	PA +!K'+PRE+DIST!+!RE, PA +NEC!

The lesson that we learned:

There may be different ways to conduct the reductions in DEL logics which require different facilities in the proof system.

Make your choice carefully! Constructing alternative semantics can help us to understand the merit of the original semantics

An alternative context-dependent semantics

The context-dependent semantics gives us a lot more freedom in designing the semantics for dynamic epistemic logic. The updates will only change the context, not the model:

$\mathcal{M}, s \Vdash \phi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_{\top} \phi$
$\mathcal{M}, s \Vdash_{\chi} \top$	\Leftrightarrow	always
$\mathcal{M}, s \Vdash_{\chi} p$	\Leftrightarrow	$p \in V(s)$
$\mathcal{M}, s \Vdash_{\chi} \neg\phi$	\Leftrightarrow	$\mathcal{M}, s \not\Vdash_{\chi} \phi$
$\mathcal{M}, s \Vdash_{\chi} \phi \wedge \psi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_{\chi} \phi$ and $\mathcal{M}, s \Vdash_{\chi} \psi$
$\mathcal{M}, s \Vdash_{\chi} \Box_i \psi$	\Leftrightarrow	$\forall t : (s \rightarrow_i t \text{ and } \mathcal{M}, t \Vdash_{\top} \chi) \text{ implies } \mathcal{M}, t \Vdash_{\chi} \psi$
$\mathcal{M}, s \Vdash_{\chi} [\psi]\phi$	\Leftrightarrow	$\mathcal{M}, s \Vdash_{\chi} \psi \text{ implies } \mathcal{M}, s \Vdash_{\chi \wedge [\chi]} \phi$

We can show: $\mathcal{M}, s \models \phi \iff \mathcal{M}, s \Vdash \phi$.

Similar semantics leads to a Gentzen-style sequent system for PAL (Maffezioli and Negri 11).

The second question is about reduction axioms: must-do or coincidence?

Can we give meaningful axiomatizations without those reduction axioms and the reduction proof method?

Yes, we can!

We will give a new axiomatization with a general proof method inspired by Epistemic Temporal Logic.

Let us go back to the standard method in normal modal logic.

To show the strong completeness: for each consistent set of formulas we need to find a model.

A *canonical model* based proof w.r.t. a normal modal logic on a class \mathbb{C} of structures usually consists of the following steps (find a model for each consistent set):

- 1 Prove the *Lindenbaum*-like lemma: every consistent set of formulas can be extended into a maximal consistent set (MCS).
- 2 Construct the canonical model based on MCSs.
- 3 Prove the *Truth Lemma*: a formula is true at a state in the canonical model iff it is in the state (an MCS).
- 4 Show that the canonical model is indeed based on some structure in \mathbb{C} .

Canonical Kripke model is the tuple $(S^c, \rightarrow^c, V^c)$ where:

- S^c is the set of all the maximal consistent set w.r.t. some proof system.
- $s \rightarrow^c t$ iff (for all ϕ : $\phi \in t$ implies $\diamond\phi \in s$)
- $V^c(p) = \{s \mid p \in s\}$

The only hard part is the truth lemma.

In an inductive proof on the structure of the formulas, how to show:

$$\mathcal{M}^c, s \models [\psi]\phi \iff [\psi]\phi \in s$$

What if we consider $[\psi]$ as a standard modality?

Definition (Extended model)

An *extended (Kripke) model* \mathcal{M} for PAL is a tuple

$(S, \rightarrow, \{\overset{\psi}{\rightarrow} \mid \psi \in \text{PAL}\}, V)$ where:

- (S, \rightarrow, V) is a standard Kripke model for PAL.
- For each ψ , $\overset{\psi}{\rightarrow}$ is a (possibly empty) binary relation over S .

We call (S, \rightarrow, V) the *Kripke core* of \mathcal{M} (notation \mathcal{M}^-).

$\mathcal{M}, s \Vdash \top$	\Leftrightarrow	always
$\mathcal{M}, s \Vdash p$	\Leftrightarrow	$s \in V(p)$
$\mathcal{M}, s \Vdash \neg\phi$	\Leftrightarrow	$\mathcal{M}, s \not\Vdash \phi$
$\mathcal{M}, s \Vdash \phi \wedge \psi$	\Leftrightarrow	$\mathcal{M}, s \Vdash \phi$ and $\mathcal{M}, s \Vdash \psi$
$\mathcal{M}, s \Vdash \Box\psi$	\Leftrightarrow	$\forall t : s \rightarrow t$ implies $\mathcal{M}, t \Vdash \psi$
$\mathcal{M}, s \Vdash [\psi]\phi$	\Leftrightarrow	$\forall t : s \overset{\psi}{\rightarrow} t$ implies $\mathcal{M}, t \Vdash \phi$

We can also build the extended canonical model:

Definition (Extended canonical model)

The extended canonical model \mathcal{M}_+^c for a normal modal logic proof system \mathbf{S} is a tuple $(S^c, \rightarrow^c, \{\overset{\psi}{\rightarrow} \mid \psi \in \text{PAL}\}, V^c)$ where:

- $(S^c, \rightarrow^c, V^c)$ is a standard canonical model for \mathbf{S} ,
- $s \overset{\psi}{\rightarrow} t$ iff for all ϕ : $\phi \in t$ implies $\langle \psi \rangle \phi \in s$.

It is easy to see that the Kripke core of \mathcal{M}_+^c is just the standard canonical model, namely $(\mathcal{M}_+^c)^- = \mathcal{M}^c$.

We can show the truth lemma w.r.t. \Vdash :

$$\mathcal{M}_+^c, s \Vdash \phi \iff \phi \in s$$

We can obtain the desired truth lemma if we can show:

$$\mathcal{M}_+^c, s \Vdash \phi \iff (\mathcal{M}_+^c)^-, s \vDash \phi \quad (\star)$$

However, \vDash and \Vdash usually do not coincide on extended models.
We hope they do coincide on $\mathcal{M}_+^c \dots$

Now let's see under what conditions the two indeed coincide.

$$\mathcal{M}, s \Vdash \phi \iff \mathcal{M}^-, s \vDash \phi$$

The hard part is again the clause for $[\psi]\phi \dots$

- ψ is true at s iff there is a unique ψ -transition at s
- if $s \xrightarrow{\psi} t$ then $\mathcal{N}^-|_{\psi}, s \Leftrightarrow \mathcal{N}^-, t$: turning dynamics into statics.

Definition (Bisimulation)

A binary relation Z is called a *bisimulation* between two pointed Kripke models \mathcal{M}, s and \mathcal{N}, t , if sZt and whenever wZv the following hold:

Invariance $p \in V^{\mathcal{M}}(w)$ iff $p \in V^{\mathcal{N}}(v)$,

Zig if $w \rightarrow w'$ for some w' in \mathcal{M} then there is a $v' \in S^{\mathcal{N}}$ with $v \rightarrow v'$ and $w'Zv'$,

Zag if $v \rightarrow v'$ for some v' in \mathcal{N} then there is a $w' \in S^{\mathcal{M}}$ with $w \rightarrow w'$ and $w'Zv'$.

Definition (Normal extended Kripke model)

An extended model $\mathcal{M} = (S, \rightarrow, \{\overset{\psi}{\rightarrow} \mid \psi \in \text{PAL}\}, V)$ for PAL is called *normal* if the following properties hold for any s, t in \mathcal{M} :

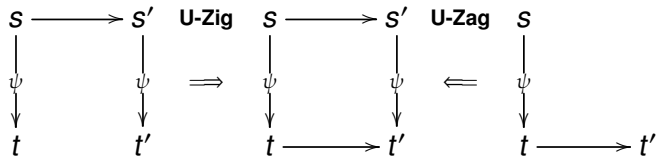
U-Functionality For any PAL formula ψ : If $\mathcal{M}, s \Vdash \psi$, then s has an unique ψ -successor. If $\mathcal{M}, s \not\vdash \psi$ then s has no outgoing ψ -transition.

U-Invariance if $s \overset{\psi}{\rightarrow} t$ then for all $p \in \mathbf{P}$:
 $s \in V(p) \iff t \in V(p)$.

U-Zig if $s \rightarrow s'$, $s' \overset{\psi}{\rightarrow} t'$ and $s \overset{\psi}{\rightarrow} t$ then $t \rightarrow t'$.

U-Zag if $s \overset{\psi}{\rightarrow} t$ and $t \rightarrow t'$ then there exists an s' such that $s \rightarrow s'$ and $s' \overset{\psi}{\rightarrow} t'$.

An extended model is called ψ -*normal* if it enjoys the last 3 properties and has functionality property for a particular ψ (ψ -Functionality).



Lemma

Given a PAL formula ψ and a ψ -normal extended Kripke model \mathcal{M} . We have

$$\mathcal{M}^- \upharpoonright_{\psi}, w \Leftrightarrow \mathcal{M}^-, v$$

if the following two conditions hold:

- 1 $w \xrightarrow{\psi} v$ in \mathcal{M} ,
- 2 for every point u in \mathcal{M} , $\mathcal{M}^-, u \vDash \psi \iff \mathcal{M}, u \Vdash \psi$.

Theorem

For any PAL formula ϕ and any normal extended Kripke model \mathcal{M} :

$$\mathcal{M}, s \Vdash \phi \iff \mathcal{M}^-, s \vDash \phi$$

Now we just need to make sure that our axioms can force the extended canonical model to be normal.

System PAN

Axiom Schemas

TAUT all the instances of tautologies

DISTK $\Box(\phi \rightarrow \chi) \rightarrow (\Box\phi \rightarrow \Box\chi)$ DIST! $[\psi](\phi \rightarrow \chi) \rightarrow ([\psi]\phi \rightarrow [\psi]\chi)$ INV $(p \rightarrow [\psi]p) \wedge (\neg p \rightarrow [\psi]\neg p)$ FUNC $\langle\psi\rangle\phi \leftrightarrow (\psi \wedge [\psi]\phi)$ NM $\Diamond\langle\psi\rangle\phi \rightarrow [\psi]\Diamond\phi$ PR $\langle\psi\rangle\Diamond\phi \rightarrow \Diamond\langle\psi\rangle\phi$

Rules

MP

$$\frac{\phi, \phi \rightarrow \psi}{\psi}$$

NECK

$$\frac{\psi}{\phi}$$

NEC!

$$\frac{\Box\phi}{\phi}$$

$$\frac{\phi}{[\psi]\phi}$$

where $p \in \mathbf{P} \cup \{\mathbf{T}\}$

Lemma

\mathcal{M}_+^c is a normal extended Kripke model.

Lemma

For any PAL formula ϕ and a point s in \mathcal{M}^c :

$$\phi \in s \iff \mathcal{M}^c, s \vDash \phi$$

Theorem

PAN is sound and strongly complete w.r.t. the standard semantics of PAL on the class of all Kripke frames.

The proof strategy consists of:

- 1 Construct an extended canonical model with update transitions.
- 2 Show that the truth lemma holds under auxiliary semantics.
- 3 Establish the equivalence between the standard semantics and the auxiliary semantics on the extended canonical model by using axioms that define the updates.
- 4 Finally we obtain the truth lemma w.r.t. the standard semantics and standard canonical model and completeness follows easily.

Actually, the uniqueness in functionality is not needed in the proof (though it eases the proof), and correspondingly, we just need $\langle \psi \rangle \top \leftrightarrow \psi$ in the system instead of $\langle \psi \rangle \phi \leftrightarrow (\psi \wedge [\psi] \phi)$: each instance of $\langle \psi \rangle \phi \rightarrow [\psi] \phi$ is provable.

The general strategy

Find a class of two-dimensional models \mathbb{C} and show the following:

$$\vDash \phi \implies \mathbb{C} \Vdash \phi \implies \vdash_{\mathbf{PAN}} \phi.$$

Same language, two logics: $\langle \mathbf{PAL}, \mathbb{M}, \vDash \rangle$ and $\langle \mathbf{PAL}, \mathbb{C}, \Vdash \rangle$.

Step 1 (Flatten the dynamics):

If $w \xrightarrow{\psi} v$ in an extended model \mathcal{N} , then $\mathcal{N}^-|_{\psi}, w \Leftrightarrow \mathcal{N}^-, v$.

Step 2:

For any ϕ and any normal \mathcal{N}, s : $\mathcal{N}, s \Vdash \phi \iff \mathcal{N}^-, s \vDash \phi$.

Step 3: $\vDash \phi \implies \mathbb{C} \Vdash \phi$ (actually: $\vDash \phi \iff \mathbb{C} \Vdash \phi$).

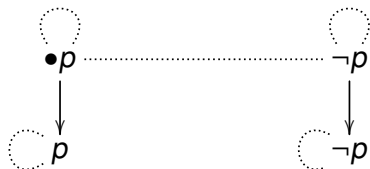
Step 4: $\mathbb{C} \Vdash \phi \iff \vdash_{\mathbf{PAN}} \phi$.

Finally: $\vDash \phi \iff \mathbb{C} \Vdash \phi \iff \vdash_{\mathbf{PAN}} \phi$.

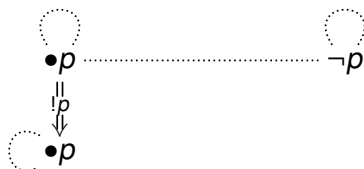
Connection with ETL

	language	model	semantics
ETL	time+K	temporal+epistemic	Kripke-like
DEL	K+events	epistemic	Kripke+ <i>dynamic</i>

$$\neg Kp \wedge EF Kp$$



$$\neg Kp \wedge [!p]Kp$$



Iterated updating epistemic structures generates special ETL-style “super models” [van Benthem et al., 2009].

The crucial axioms

PR is in the shape of $\langle a \rangle \diamond \phi \rightarrow \diamond \langle a \rangle \phi$ (or $\Box [a] \phi \rightarrow [a] \Box \phi$).

NM is in the shape of $\diamond \langle a \rangle \phi \rightarrow [a] \diamond \phi$ (or $\langle a \rangle \Box \phi \rightarrow \Box [a] \phi$).

No Learning (NL) in ETL: $\diamond \langle a \rangle \phi \rightarrow \langle a \rangle \diamond \phi$ (or $[a] \Box \phi \rightarrow \Box [a] \phi$).

Note the **difference** between NM and NL:

$$\diamond \langle a \rangle \phi \rightarrow [a] \diamond \phi \text{ (NM) vs. (NL) } \diamond \langle a \rangle \phi \rightarrow \langle a \rangle \diamond \phi$$

NL is too strong: if you consider possible that an event is executable then it must be executable (take ϕ to be \top).

One secret of PAL (and DEL in general) is the *no miracles*-like axiom/property: You can only learn by observation.

We can give up the axioms of INV and FUNC with no problems [Wang and Li, 2012].

Our axiomatization can help to explain many recent results about PAL or other dynamic epistemic logics:

- An explanation to the “reduction phenomena”.
- The axiomatization of the “substitution core” of PAL as in [Holliday et al., 2012].
- The representation results between action model DEL and ETL as in [van Benthem et al., 2009] and [Dégremont et al., 2011].
- The characterization result of partial p-morphism as in [van Benthem, 2012].

It also shows that: logic (validities) cannot distinguish DEL and special ETL! It is about two different perspectives: local vs. global.

What kind of global properties can be constructed by local constructions?

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$K_i(\phi \rightarrow \psi) \rightarrow (K_i\phi \rightarrow K_i\psi)$
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!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \rightarrow \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]K_i\phi \leftrightarrow (\psi \rightarrow K_i(\psi \rightarrow [\psi]\phi))$
Rules	
NECK	$\frac{\phi}{K_i\phi}$
MP	$\frac{\phi, \phi \rightarrow \psi}{\psi}$
Your selection	
DIST!	$[\psi](\phi \rightarrow \chi) \rightarrow ([\psi]\phi \rightarrow [\psi]\chi)$
NEC!	$\frac{\phi}{[\psi]\phi}$
RE	$\frac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$

There are also many new questions

- Can you axiomatize the substitution core of PAL (the collection of valid formulas which are closed under uniform substitution)? [Holliday et al., 2012]
- Can you characterize (syntactically) the “successful” fragment of PAL? [Holliday and Icard III., 2010]
- What operations can be defined by reduction axioms? [van Benthem, 2012]
- Three-value semantics of PAL. [Dechesne et al., 2008]

PAL with natural extensions:

- Quantifying over announcements. [Ågotnes et al., 2009]
- PAL with protocols. [Wang, 2011]
- PAL with agent types. [Liu and Wang, 2013]
- With common knowledge: more expressive than modal logic [van Benthem et al., 2006]. A expressiveness hierarchy: [Zou, 2012]
- With iterations: undecidable on the class of arbitrary models [Moss and Miller, 2005], but decidable on single-agent S5. [Ding, 2014]
- Model checking PAL with all these extras.

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