Logical Dynamics II: Public Announcement Logic

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A very brief pre-history

[Stalnaker, 1978] on *assertion*:

- Its content is *dependent* on its context.
- It *modifies* the context.

These points inspired the invention of *dynamic semantics* [Groenendijk and Stokhof, 1991] and *update semantics* [Veltman, 1996]:

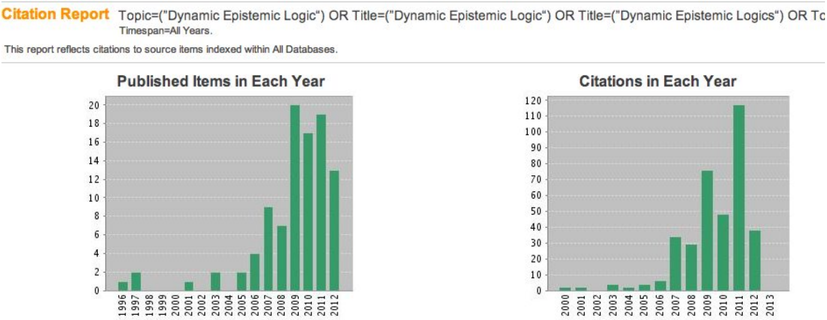
The meaning of a sentence is identified with its *context change potential*.

One step further:

The meaning of a communicative event is the *change* it brings to the epistemic states of the participants in the discourse.

- [Gerbrandy and Groeneveld, 1997] combined the ideas of [Veltman, 1996] and [Fagin et al., 1995]: dynamic epistemic semantics for announcements.
- [Gerbrandy, 1999] developed the idea further. Some ILLC students rediscovered [Plaza, 1989] in which the public announcement logic (PAL) was proposed.
- [Baltag et al., 1998] proposed the dynamic epistemic logic with action model updates.

In this century



Overview books:

- Dynamic Epistemic Logic [van Ditmarsch et al., 2007]
- Logical Dynamics of Information and Interaction [van Benthem, 2011]
- Johan van Benthem on Logic and Information Dynamics

Discipline

From `Springer Link`

Computer Science	384
Philosophy	280
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Subdiscipline

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Logic and Philosophy of Language	251
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Let's go back to the origin...

Do we really understand thoroughly what we are doing? What is *Dynamic Epistemic Logic* as a field?

In searching for the answer, let us go back to the basics.

We will focus on axiomatizations (of PAL):

- It helps us to understand the semantics-driven logics better.
- It helps to compare with related approaches.
- It may direct the future developments of the field.

Public Announcement Logic (PAL)

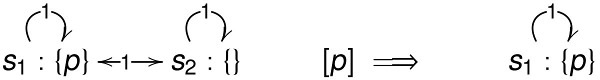
The language of *Public Announcement Logic (PAL)*:

$$\phi \quad ::= \quad \top \mid p \mid \neg\phi \mid (\phi \wedge \phi) \mid \Box_i\phi \mid [\phi]\phi \text{ (also write } [!\phi]\phi)$$

It is interpreted on (**S5**) Kripke models $\mathcal{M} = (S, \{\rightarrow_i\}_{i \in I}, V)$:

$$\boxed{\begin{array}{lcl} \mathcal{M}, s \vDash \Box_i\psi & \Leftrightarrow & \forall t : s \rightarrow_i t \implies \mathcal{M}, t \vDash \psi \\ \mathcal{M}, s \vDash [\psi]\phi & \Leftrightarrow & \mathcal{M}, s \vDash \psi \text{ implies } \mathcal{M}|_\psi, s \vDash \phi \end{array}}$$

where $\mathcal{M}|_\psi = (S', \{\rightarrow'_i \mid i \in \mathbf{I}\}, V')$ such that: $S' = \{s \mid \mathcal{M}, s \vDash \psi\}$, $\rightarrow'_i = \rightarrow_i |_{S' \times S'}$ and $V'(p) = V(p) \cap S'$.



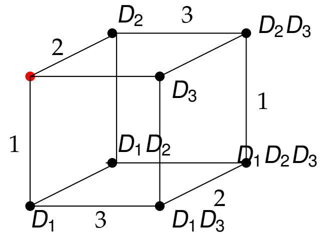
$$\mathcal{M}, s_1 \vDash \neg\Box_1 p \wedge [p]\Box_1 p$$

The classic example

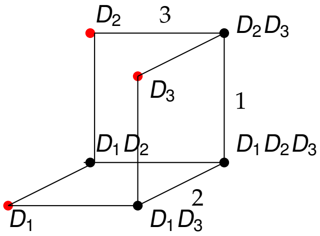
Muddy Children - the setting

- Out of n children, $k \geq 1$ got mud on their faces while playing.
- They can see whether other kids are dirty, but there is no mirror for them to discover whether they are dirty themselves.
- Then father walks in and states: "At least one of you is dirty!" Then he requests "If you know you are dirty, step forward now."
- If nobody steps forward, he repeats his request: "If you now know you are dirty, step forward now."
- After exactly k requests to step forward, the k dirty children suddenly do so (assuming they are honest and perfect reasoners).

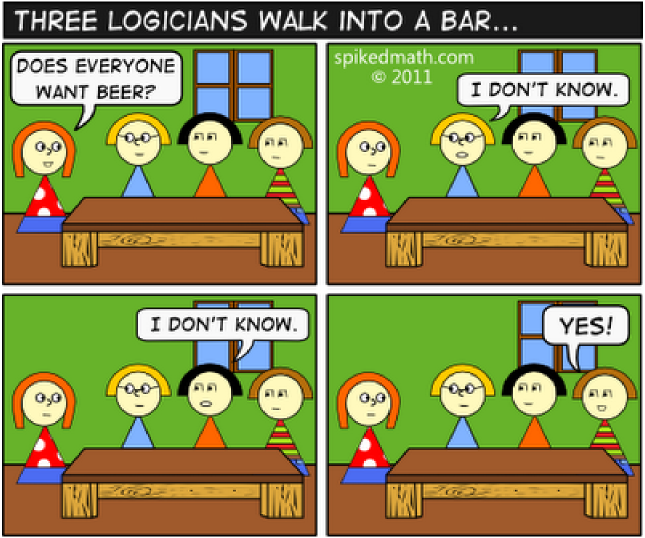
When there are 3 dirty children...



"At least one of you is dirty!"
Announcement: $\psi = D_1 \vee D_2 \vee D_3$



Yet another example



Public Announcement Logic (PAL)

The classic modal logic questions:

- Do we have a complete axiomatization?
- Do we have complete axiomatizations w.r.t. certain classes of frames ?
- Do the axioms and rules of a normal modality also hold for $[\psi]$?
- Is PAL invariant under bisimulation or other equivalence notions?
- Can we define an equivalent game semantics?
- Is there a correspondence between the finite fragment of it and n-bisimulation in some way?
- Does it have finite model property?
- Is it decidable?
- How is its definability over models and frames?
- What if we add announcement operators on propositional logic?
- What is the relationship between PAL and modal (epistemic) logic?
- Is it translatable into first-order logic?

Get familiar with it first!

Try to get a feeling of the semantics of PAL by checking the validity of the following formula schemas and rules.

- $\langle\phi\rangle\psi \to [\phi]\psi$, $\langle\phi\rangle\psi \to \phi$, $\langle\phi\rangle\psi \leftrightarrow (\phi \wedge [\phi]\psi)$
- $[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$, $[\psi](\phi \to \chi) \leftrightarrow ([\psi]\phi \to [\psi]\chi)$
- $[\psi]p \leftrightarrow (\psi \to p)$, $[\psi]\neg\phi \leftrightarrow (\psi \to \neg\phi)$ (×), $[\psi]\neg\phi \leftrightarrow \neg[\psi]\phi$ (×), $[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
- $\dfrac{\phi}{[\psi]\phi}$, $\dfrac{\phi(p)}{\phi(\psi)}$ (×), $\dfrac{\phi \leftrightarrow \psi}{[\phi]\chi \leftrightarrow [\psi]\chi}$, $\dfrac{\phi \leftrightarrow \psi}{[\chi]\phi \leftrightarrow [\chi]\psi}$
- $[\psi]\Box_i\phi \leftrightarrow (\psi \to \Box_i(\psi \to [\psi]\phi))$, $[\psi]\Box_i\phi \leftrightarrow (\psi \to \Box_i[\psi]\phi)$
- $[\psi][\chi]\phi \leftrightarrow [\psi \wedge \chi]\phi$ (×), $[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
- $[\psi]\Box_i\psi$ (×)

Basic System **PA**: Axioms and Rules

Different proof systems were proposed in the literature which share the following axiom schemas and rules.

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\Box_i(\phi \to \psi) \to (\Box_i\phi \to \Box_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]\Box_i\phi \leftrightarrow (\psi \to \Box_i(\psi \to [\psi]\phi))$
Rules	
NECK	$\dfrac{\phi}{\Box_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$

No uniform substitution!

Axioms and Rules

Axiom Schemas	
DIST!	$[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
!K'	$[\psi]\Box_i\phi \leftrightarrow (\psi \to \Box_i[\psi]\phi)$
Rules	
NEC!	$\dfrac{\phi}{[\psi]\phi}$
RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$

Reduction /recursion axioms

Axiom Schemas	
!ATOM	$[\psi]p \leftrightarrow (\psi \rightarrow p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \rightarrow \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]\Box_i\phi \leftrightarrow (\psi \rightarrow \Box_i(\psi \rightarrow [\psi]\phi))$

Semantic update as syntactic relativization:

$$\mathcal{M}|_\psi, s \models \phi \iff \mathcal{M}, s \models (\phi)^\psi$$

Soundness and Completeness

Proposition

All the above axiom schemas and rules are sound w.r.t to the standard $\mathtt{PAL}$ semantics.

Theorem ([Plaza, 1989])

$\mathtt{PAL}$ is equally expressive as basic modal logic.

$$
\begin{array}{lllllll}
t(\top) & = & \top & t([\psi]\top) & = & t(\psi \to \top) \\
t(p) & = & p & t([\psi]p) & = & t(\psi \to p) \\
t(\neg\phi) & = & \neg t(\phi) & t([\psi]\neg\phi) & = & t(\psi \to \neg[\psi]\phi) \\
t(\phi_1 \wedge \phi_2) & = & t(\phi_1) \wedge t(\phi_2) & t([\psi](\phi_1 \wedge \phi_2)) & = & t([\psi]\phi_1 \wedge [\psi]\phi_2) \\
t(\Box_i\phi) & = & \Box_i t(\phi) & t([\psi]\Box_i\phi) & = & t(\psi \to \Box_i(\psi \to [\psi]\phi)) \\
 & & & t([\psi][\chi]\phi) & = & t([\psi]t([\chi]\phi))
\end{array}
$$

We can obtain another translation t' by revising t: just replace the last item by $t'([\psi][\chi]\phi) = t'([\psi \wedge [\psi]\chi]\phi)$

PAL is equally expressive as basic modal logic

Intuitively, the translation "pushes" the $[\cdot]$ modality through the formula to the inner part. How to prove that the translation indeed produces $[\cdot]$-free formulas?

Definition (Complexity of PAL formulas)

$$\begin{array}{lcl} c(\top) & = & 1 \\ c(p) & = & 1 \\ c(\neg\phi) & = & 1 + c(\phi) \\ c(\phi_1 \wedge \phi_2) & = & 1 + max(c(\phi_1), c(\phi_2)) \\ c(\Box_i\phi) & = & 1 + c(\phi) \\ c([\psi]\phi) & = & (5 + c(\psi)) \cdot c(\phi) \end{array}$$

PAL is equally expressive as modal logic

We can show that:

$c(\phi) > c(\psi)$		If ψ is a proper subformula of ϕ
$c([\psi]\top)$	$>$	$c(\psi \to \top)$
$c([\psi]p)$	$>$	$c(\psi \to p)$
$c([\psi]\neg\phi)$	$>$	$c(\psi \to \neg[\psi]\phi)$
$c([\psi](\phi_1 \land \phi_2))$	$>$	$c([\psi]\phi_1 \land [\psi]\phi_2)$
$c([\psi]\Box_i\phi)$	$>$	$c(\psi \to \Box_i(\psi \to [\psi]\phi))$
$c([\psi][\chi]\phi)$	$>$	$c([\psi \land [\psi]\chi]\phi)$
$c([\psi][\chi]\phi)$	$>$	$c([\psi]t([\chi]\phi))$

PAL is equally expressive as modal logic

We can prove by induction on the **complexity** of ϕ that (cf. DEL book Lemma 7.22, 7.23):

Proposition

$t(\phi)$ and $t'(\phi)$ are $[\cdot]$-free.

We can show that:

Proposition

$\models \phi \leftrightarrow t(\phi)$ *and* $\models \phi \leftrightarrow t'(\phi)$

Is $t(\phi) = t'(\phi)$?

Completeness via Reduction

Completeness is proved via reduction and the completeness of basic modal logic **K**:

$$\models \phi \implies \models t(\phi) \overset{\text{comp. of }\mathbf{K}}{\implies} \vdash_{\mathbf{K}} t(\phi) \implies \vdash_{\mathbf{PA}^+} t(\phi) \overset{\mathit{Rd.Axioms}}{\implies} \vdash_{\mathbf{PA}^+} \phi$$

We can mimic t and t' in proof systems stronger than **PA**.

Proposition

$\vdash_{\mathbf{PA}+\mathtt{RE}} \phi \leftrightarrow t(\phi)$ *and* $\vdash_{\mathbf{PA}+!\mathtt{COM}} \phi \leftrightarrow t'(\phi)$

Theorem ([Plaza, 1989])

$\mathbf{PA}+\mathtt{RE}$ is complete w.r.t. the standard semantics of $\mathtt{PAL}$.

Theorem (cf. e.g., [van Ditmarsch et al., 2007])

$\mathbf{PA}+!\mathtt{COM}$ is complete w.r.t. the standard semantics of $\mathtt{PAL}$.

Public Announcement Logic (PAL)

Now we can answer most of the following questions:

- * Do we have a complete axiomatization?
- * Do we have complete axiomatizations w.r.t. other classes of frames?
- * Do the axioms and rules for $\Box$ also hold for $[\psi]$?
- * Is PAL invariant under bisimulation?
- Can we define an equivalent game semantics?
- Is it translatable into first-order logic?
- Is there a correspondence between a finite fragment of PAL and n-bisimulation in some way?
- * Does it have finite model property?
- * Is it decidable?
- * How is its definability power (over models and frames)?
- *What if we add announcement operators on propositional logic?

Now we define the following translation t_x (t_y is omitted) from formulas of PAL to formulas in FOL (only two variables are enough, just like the case for modal logic):

$$
\begin{array}{ccc}
t_x(\phi) & = & t_x^{\top}(\phi) \\
t_x^{\psi}(\top) & = & \top \\
t_x^{\psi}(p) & = & Px \\
t_x^{\psi}(\neg\phi) & = & \neg t_x^{\psi}(\phi) \\
t_x^{\psi}(\phi_1 \wedge \phi_2) & = & t_x^{\psi}(\phi_1) \wedge t_x^{\psi}(\phi_2) \\
t_x^{\psi}(K\phi) & = & \forall y : (xRy \text{ and } t_y^{\top}(\psi)) \text{ implies } t_y^{\psi}(\phi) \\
t_x^{\psi}([\chi]\phi) & = & t_x^{\psi}(\chi) \to t_x^{\psi\wedge[\psi]\chi}(\phi)
\end{array}
$$

Need to show that 1. the above rewriting (view from left to right) will terminate and 2. the translation is faithful.
For 2, NTS: $\mathcal{M}, s \vDash \phi \iff \mathcal{M} \Vdash t_x(\phi)[s]$.

Reduction? So what?

Theorem ([Lutz, 2006])

PAL is exponentially more succinct than modal logic on arbitrary models.

$\phi_0 = \top$ and $\phi_{i+1} = \langle\langle\phi_i\rangle\Diamond_1\top\rangle\Diamond_2\top$.

Theorem ([French et al., 2011])

PAL is exponentially more succinct than modal logic on S5 models if there are more than 3 agents.

The 2-agent case is still open!

Our first question

What about the core system **PA**? Is it complete?

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\Box_i(\phi \to \psi) \to (\Box_i\phi \to \Box_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \land \chi) \leftrightarrow ([\psi]\phi \land [\psi]\chi)$
!K	$[\psi]\Box_i\phi \leftrightarrow (\psi \to \Box_i(\psi \to [\psi]\phi))$
Rules	
NECK	$\dfrac{\phi}{\Box_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$

Our first question

Now what about the core system **PA**? Is it complete?

In some published papers and books, **PA** is also mentioned as a complete system for `PAL`. Many people use the **PA**-like systems to axiomatize other dynamic epistemic logics.

Unfortunately, **PA** and many of its "close friends" are not complete, and some works are not even patchable.

The reduction technique turns out to be extremely useful in many applications and thus dominates the field of DEL.

- Logic is more than it appears!
- Update-closeness may be considered as a desired property of a logic: it shows the logic has enough pre-encoding power [van Benthem et al., 2006].
- Compositional analysis of post-conditions.
- The orthodox programme of DEL:
  static logic+dynamic operators+reduction
- Also good for lazy guys...

There are also important works beyond the reduction programme e.g., [van Benthem et al., 2009], but without alternative general method.

The second question

Can we give meaningful axiomatizations without those reduction axioms and the reduction proof method?

Yes, we can!

We will give a new axiomatization with a general proof method inspired by Epistemic Temporal Logic.

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