BEYOND "KNOWING THAT" (IV)

LOGICS OF KNOWING HOW

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Background

Plan-based knowing how

Strategy-based knowing how

Further directions

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- In epistemology:

RELEVANCE OF A FORMAL ACCOUNT OF KNOWING HOW

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    - Can knowledge-how be reduced to knowledge-that?

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    - Can an autonomous agent know how to achieve some goal?

Interesting examples in the philosophy literature

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- We do not focus on the philosophical debate between intellectualism (e.g., Stanley & Williamson 2001) and anti-intellectualism (e.g., Ryle 49). See the collection of 200^{+} papers on the topic at `philpapers.org`.

A PARTICULAR TYPE OF "KNOWING HOW"

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- We do not study "knowing how" in the following senses: I know how turtles reproduce; I know how happy she is; I know how to speak Chinese; I know how to behave at the dinner table....

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Activity directed, rule directed, goal directed, maintaining goal … see [Gochet 2013]

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- Knowing how to achieve a goal may not entail that you *can* realize the goal now: a chef knows how to make cakes even when there is no sugar. The chef can make a cake, given all the ingredients and equipments are there.

TOWARDS A LOGIC OF KNOWING HOW

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Two observations inspired by the discussions in philosophy:

- Knowing how to achieve a goal may not entail that you *can* realize the goal now: a chef knows how to make cakes even when there is no sugar. The chef can make a cake, given all the ingredients and equipments are there.
- Even when you can win a lottery by luckily buying the right ticket, it does not mean you know how to win the lottery, since you cannot knowingly guarantee the result.

Plan-based knowing how

The language is defined as follows:

$$\varphi \quad ::= \quad \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{Kh}(\varphi, \varphi)$$

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A model is simply a labeled transition system representing the (known) abilities of the agent: $(\mathcal{S}, \Sigma, \mathcal{R}, \mathcal{V})$ where:

- $\mathcal{S}$ is a non-empty set of states;
- Σ is a non-empty set of actions (not in the language!);
- $\mathcal{R} : \Sigma \to 2^{\mathcal{S} \times \mathcal{S}}$ is a collection of transitions labelled by Σ;
- $\mathcal{V} : S \to 2^{\mathsf{P}}$ is a valuation function.


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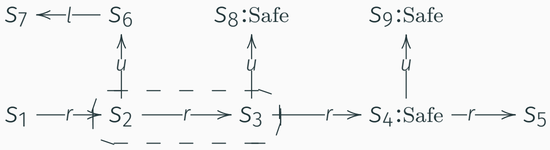
A rookie spy sneaking in an enemy building was guided by his headquarters. The communication with the HQ was lost at some point. Now someone spotted him and pulled the alarm. In panic he got lost...



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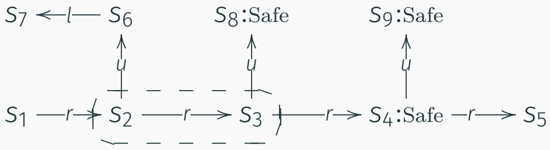

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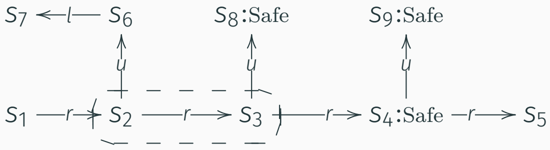

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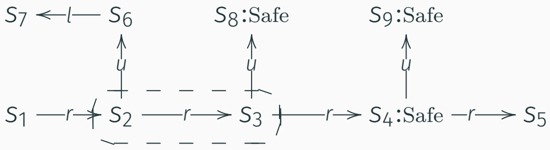

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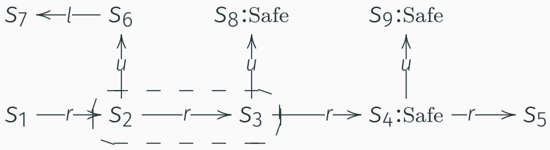

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Kh(ψ, φ) is true iff there is a plan σ (sequence of actions) such that you know that, given ψ, σ is always fully executable and it can get you to some φ world in the end.

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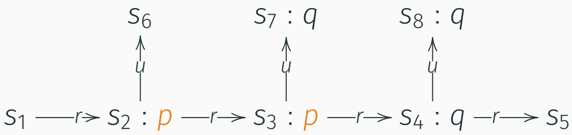

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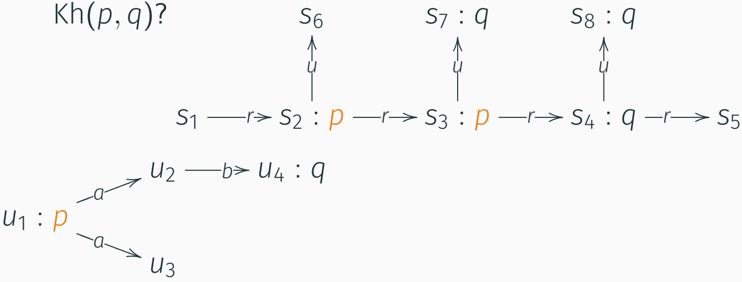

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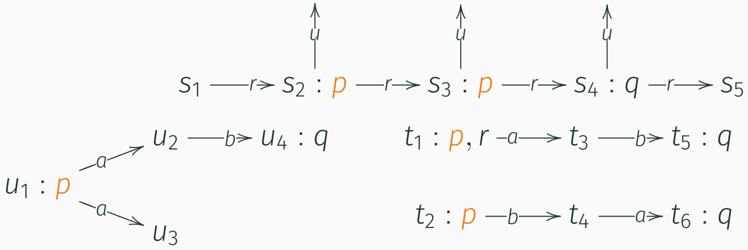

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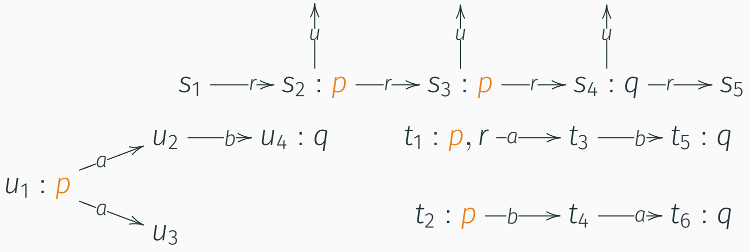

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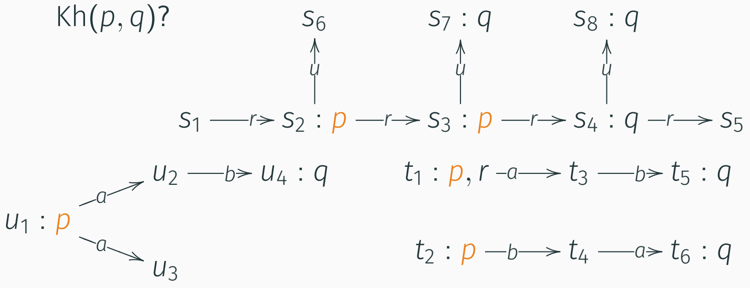

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OTHER SEMANTICS

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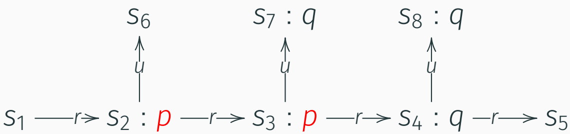
where $s \xrightarrow{\sigma}_W t$ means that the execution of σ from s may terminate at t. E.g., $\mathsf{Khw}(p, q)$ is true below.

$$t : q \xleftarrow{a} s : p \xrightarrow{a} w : \xrightarrow{a} u : q$$

DIFFERENCES OF THE THREE KNOW-HOW OPERATORS



- Kh(p, q) holds: *ru* is the only strongly executable witness.

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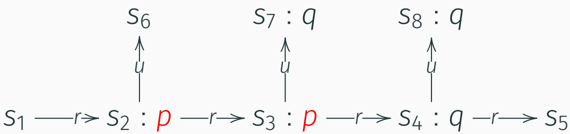

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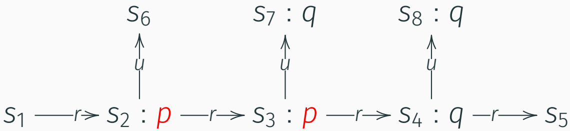

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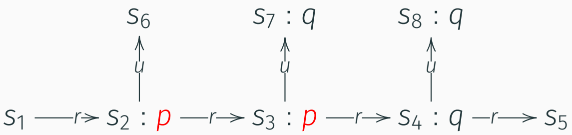

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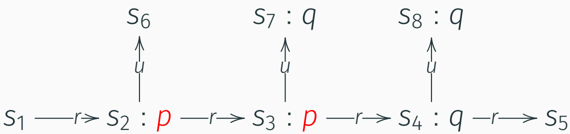

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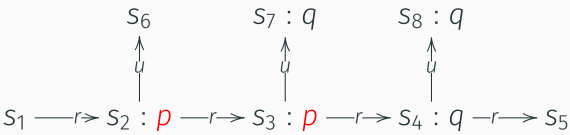

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Clearly $\mathrm{Kh}(p, q)$ can be defined by $\mathrm{Khm}(p, \top, q)$.

PROOF SYSTEM FOR THE FIRST SEMANTICS [WANG LORI15]

Axioms		Rules	
TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTU	$Ap \wedge A(p \to q) \to Aq$	NECU	$\dfrac{\varphi}{A\varphi}$
COMPKh	$Kh(p,r) \wedge Kh(r,q) \to Kh(p,q)$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
EMP	$A(p \to q) \to Kh(p,q)$		
TU	$Ap \to p$		
4KU	$Kh(p,q) \to AKh(p,q)$		
5KU	$\neg Kh(p,q) \to A\neg Kh(p,q)$		

Provable:

PREKh: $Kh(Kh(p,q) \wedge p, q)$, POSTKh: $Kh(r, Kh(p,q) \wedge p) \to Kh(r,q)$

MONO: from $\vdash \varphi \to \psi$ infer $\vdash Kh(\chi,\varphi) \to Kh(\chi,\psi)$.

PROOF SYSTEM FOR THE SECOND SEMANTICS [LI WANG ICLA17]

Axioms

TAUT	all axioms of propositional logic
DISTU	$\mathsf{A}p \wedge \mathsf{A}(p \to q) \to \mathsf{A}q$
COMPKh	$\mathsf{Kh}(p,o,r) \wedge \mathsf{Kh}(r,o,q) \wedge \mathsf{A}(r \to o) \to \mathsf{Kh}(p,o,q)$
EMP	$\mathsf{A}(p \to q) \to \mathsf{Kh}(p,\bot,q)$
TU	$\mathsf{A}p \to p$
4KU	$\mathsf{Kh}(p,o,q) \to \mathsf{AKh}(p,o,q)$
5KU	$\neg\mathsf{Kh}(p,o,q) \to \mathsf{A}\neg\mathsf{Kh}(p,o,q)$
UKhm	$\mathsf{A}(p' \to p) \wedge \mathsf{A}(o \to o') \wedge \mathsf{A}(q \to q') \wedge \mathsf{Kh}(p,o,q) \to \mathsf{Kh}(p',o',q')$
OneKhm	$\mathsf{Kh}(p,o,q) \wedge \neg\mathsf{Kh}(p,\bot,q) \to \mathsf{Kh}(p,\bot,o)$

Rules are MP, NECU, SUB as before.

PROOF SYSTEM FOR THE THIRD SEMANTICS [LI 17]

Axioms		Rules
TAUT	all axioms of propositional logic	MP
DISTU	$\mathsf{A}p \wedge \mathsf{A}(p \to q) \to \mathsf{A}q$	NECU
AKh	$\mathsf{A}(p' \to p) \wedge \mathsf{A}(q \to q') \wedge \mathsf{Khw}(p, q) \to \mathsf{Khw}(p', q')$	SUB
EMP	$\mathsf{A}(p \to q) \to \mathsf{Khw}(p, q)$	
TU	$\mathsf{A}p \to p$	
4KU	$\mathsf{Khw}(p, q) \to \mathsf{A}\mathsf{Khw}(p, q)$	
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$s_1 : p \xrightarrow{a} s_3 : r \xrightarrow{b} s_5 : q$ $\mathsf{Khw}(p,r) \wedge \mathsf{Khw}(r,q) \not\to \mathsf{Khw}(p,q)$

$s_2 : p, r \xrightarrow{b} s_4 : q$

A single canonical model does not work!

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Given a maximal consistent set Γ w.r.t. $\mathbb{SKH}$, let $\Sigma_\Gamma = \{\langle\psi,\varphi\rangle \mid \mathsf{Kh}(\psi,\varphi) \in \Gamma\}$, the canonical model for Γ is $\mathcal{M}^c_\Gamma = \langle \mathcal{S}^c_\Gamma, \mathcal{R}^c, \mathcal{V}^c \rangle$ where:

- $\mathcal{S}^c_\Gamma = \{\Delta \mid \Delta$ is a MCS w.r.t. $\mathbb{SKH}$ and $\Gamma|_{\mathsf{Kh}} = \Delta|_{\mathsf{Kh}}\}$;
- $\Delta \xrightarrow{\langle\psi,\varphi\rangle}_c \Theta$ iff $\mathsf{Kh}(\psi,\varphi) \in \Gamma, \psi \in \Delta$, and $\varphi \in \Theta$;
- $p \in V^c(\Delta)$ iff $p \in \Delta$.

Clearly Γ is a state in $\mathcal{M}^c_\Gamma$.

Lemma (Truth lemma)

For any $\varphi \in \Gamma : \mathcal{M}^c_\Gamma, \Delta \vDash \varphi \iff \varphi \in \Delta$

$\Longrightarrow$: We do not prove the contrapositive. It requires induction over the length of the witness sequence σ for the truth of Kh(ψ, φ), where `COMPKh` plays an important role.

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Theorem

The proof systems are strongly complete w.r.t. the class of all models w.r.t the corresponding semantics.

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See Yanjun Li's PhD thesis for decidability of these logics via finite canonical models.

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  For any $X \in \nu_{\mathcal{M}}(w)$, there is $X' \in \nu_{\mathcal{N}}(v)$ such that for all $x' \in X'$ there is $x \in X$ such that xZx'.

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- The "neighborhood" can be viewed as the collection of the sets that the agent can ensure to achieve by some plans.
- The extra complications are due to the fact that $\mathrm{Kh}(\psi, \varphi)$ is global.

We write $U \xrightarrow{\sigma} V$ whenever σ is strongly executable for all $u \in U$, and V is the set of states reachable from U after executing σ.

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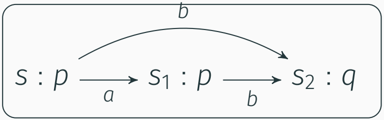

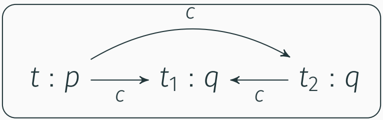

- $X \xrightarrow{\epsilon} X$ thus $X \rightarrow X$ for all subsets of the state space.
- $\{s, s_1\} \xrightarrow{b} \{s_2\}$ thus $\{s, s_1\} \rightarrow \{s_2\}$.

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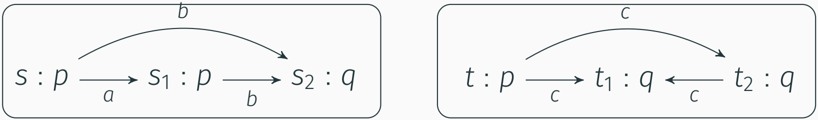

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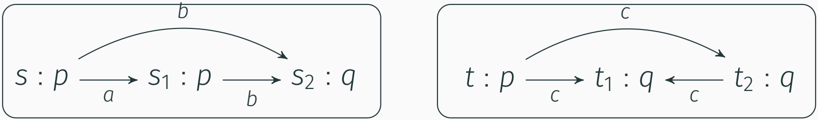

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- The above two models satisfy exactly the same Kh-formulas.

Definition (Kh-**Bis** [Fervari, Velázquez-Quesada, Wang SR17])

Let $\mathcal{M} = \langle W, \mathcal{R}, \mathcal{V} \rangle$ and $\mathcal{M}' = \langle W', \mathcal{R}', \mathcal{V}' \rangle$, a non-empty relation $Z \subseteq W \times W'$ is called an Kh-bisimulation between $\mathcal{M}$ and $\mathcal{M}'$ if and only if wZw' implies:

Atom: $\mathcal{V}(w) = \mathcal{V}'(w')$.

Kh-**Zig:** for any propositionally definable $U \subseteq W$, if $U \to V$ for some $V \subseteq W$, then there is $V' \subseteq W'$ such that
(i) $Z[U] \to V'$ and
(ii) for each $v' \in V'$ there is a $v \in V$ such that vZv'.

Kh-**Zag:** Symmetric

A-**Zig:** for any v in W there is a v' in W' such that vZv'.

A-**Zag:** Symmetric

We do need the A-Zig and A-Zag in the definition, although A is definable by Kh.

Theorem (Invariance)

Let $\mathcal{M}, w$ and $\mathcal{M}', w'$ be two pointed models, with $\mathcal{M} = \langle W, \mathcal{R}, \mathcal{V} \rangle$ and $\mathcal{M}' = \langle W', \mathcal{R}', \mathcal{V}' \rangle$. If $\mathcal{M}, w \leftrightarroweq_{\mathsf{Kh}} \mathcal{M}', w'$, then $\mathcal{M}, w \equiv_{\mathsf{Kh}} \mathcal{M}', w'$.

Theorem (Hennessy–Milner)

Let $\mathcal{M} = \langle W, \mathcal{R}, \mathcal{V} \rangle$, $\mathcal{M}' = \langle W', \mathcal{R}', \mathcal{V}' \rangle$ be two finite models, $w \in W$ and $w' \in W'$. $\mathcal{M}, w \equiv_{\mathsf{Kh}} \mathcal{M}', w'$ iff $\mathcal{M}, w \leftrightarroweq_{\mathsf{Kh}} \mathcal{M}', w'$.

Definition

Let $\mathcal{M} = \langle W, \mathcal{R}, \mathcal{V}\rangle$ and $\mathcal{M}' = \langle W', \mathcal{R}', \mathcal{V}'\rangle$ be two relational models. A non-empty relation $Z \subseteq (W \times W')$ is called an Khm-bisimulation between $\mathcal{M}$ and $\mathcal{M}'$ if and only if wZw' implies:

Atom, U-Zig and U-Zag as before.

Khm-**Zig:** for any propositional definable $U \subseteq W$, if $U \xrightarrow{X} V$ for some $X, V \subseteq W$, then there are $X', V' \subseteq W'$ such that
(i) $Z[U] \xrightarrow{X'} V'$,
(ii) for each $x' \in X'$ there is a $x \in X$ such that xZx',
(iii) for each $v' \in V'$ there is a $v \in V$ such that vZv'.

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A non-empty relation $Z \subseteq (W \times W')$ is called an Khw-bisimulation if wZw' implies:

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Khm-**Zig** for any propositional definable $U \subseteq W$, if $U \rightarrow_W V$ for some $V \subseteq W$, then
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Khm-**Zag** Symmetric.

The corresponding invariance results and Hennessy–Milner-like theorems hold.

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The corresponding invariance results and Hennessy–Milner-like theorems hold.

The logic of Khm is strictly more expressive than the logic of Kh but incomparable with the logic of Khw.

A BRIEF SUMMARY FOR NOW

Features of the logics of *knowing how* so far:

- Global knowledge
- Conditional modality
- No explicit *knowing that* operator
- Based on linear plans
- No observation during plan executions

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What about a local notion with explicit know-that operator?

Strategy-based knowing how

LANGUAGE AND MODELS [FERVARI, HERZIG, LI, WANG IJCAI17]

$$\varphi := p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{K}\varphi \mid \mathsf{Kh}\varphi$$

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Note that we have the explicit know-that operator K in the language.

language and models [Fervari, Herzig, Li, Wang IJCAI17]

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A model is a labeled transition system with an epistemic relation: $\langle \mathcal{S}, \Sigma, \mathcal{R}, \sim, \mathcal{V} \rangle$ where:

- $\langle \mathcal{S}, \Sigma, \mathcal{R}, \mathcal{V} \rangle$ is a labelled transition system as before.
- $\sim \subseteq S \times S$ is an equivalence relation (bubbles everywhere).

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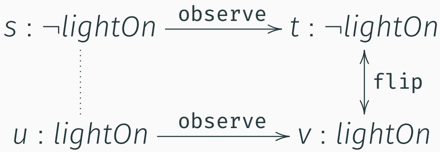
Example (reflexive arrows are omitted)



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UNIFORMLY EXECUTABLE STRATEGY

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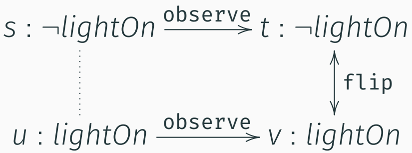

$\sigma' = \{\{s, u\} \mapsto \texttt{observe}, \{t\} \mapsto \texttt{flip}\}$ is uniformly executable.

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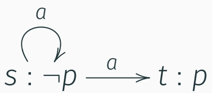
Example (M is depicted as follows)



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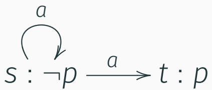


$\sigma = \{\{s\} \mapsto a\}$ is uniformly executable, but there is an infinite execution of σ starting from s. $\mathcal{M}, s \not\models \mathrm{Kh}p$.

A COMPLETE AXIOMATIZATION

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$Kp \wedge K(p \to q) \to Kq$	NECK	$\dfrac{\varphi}{K\varphi}$
T	$Kp \to p$	MonoKh	$\dfrac{\varphi \to \psi}{Kh\varphi \to Kh\psi}$
4	$Kp \to KKp$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg Kp \to K\neg Kp$		
AxKtoKh	$Kp \to Khp$		
AxKhtoKhK	$Khp \to KhKp$		
AxKhtoKKh	$Khp \to KKhp$		
AxKhKh	$KhKhp \to Khp$		
AxKhbot	$\neg Kh\bot$		

PROPERTIES

- Kh is not normal

$$\not\models \mathrm{Kh}p \wedge \mathrm{Kh}(p \to q) \to \mathrm{Kh}q$$

- negative introspection provable:

$$\models \neg\mathrm{Kh}p \to \mathrm{K}\neg\mathrm{Kh}p$$

- sequences of modal operators reduce:

$$\models \mathrm{KK}p \leftrightarrow \mathrm{K}p$$

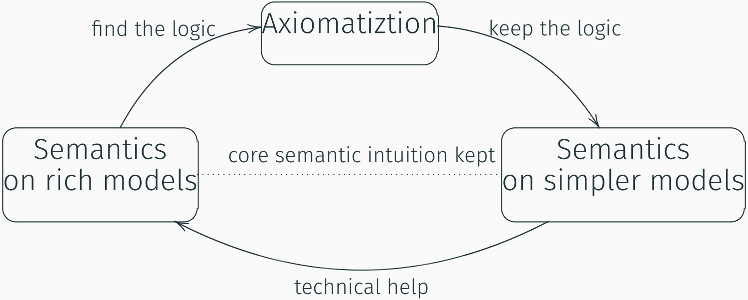
$$\models \mathrm{KhKh}p \leftrightarrow \mathrm{Kh}p$$

$$\models \mathrm{KhK}p \leftrightarrow \mathrm{Kh}p$$

$$\models \mathrm{KKh}p \leftrightarrow \mathrm{Kh}p$$

- sound and complete: soundness of $\mathrm{KhKh}p \to \mathrm{Kh}p$ is highly non-trivial!
- decidable: we can construct a finite canonical model.

AGAIN, WE ARE SEEKING FOR A SIMPLIFIED SEMANTICS



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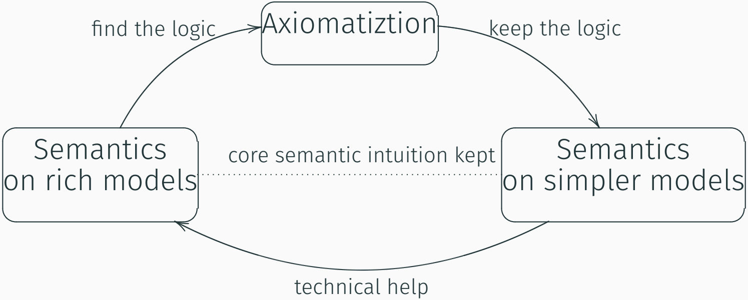

ALTERNATIVE NEIGHBOURHOOD SEMANTICS [ONGOING]

The model class $\mathcal{C}$ consists of all *mixed* epistemic models $\langle S, \sim, N, V \rangle$ satisfying the following conditions.

- For all $s \in W$, any $X, Y \subseteq W, X \in N(s)$ implies $Y \in N(s)$ (`MonoKh`).
- For all $s \in W$, $\emptyset \notin N(s)$ (`AxKhbot`)
- For any $s, t \in W$, $s \sim t$ implies $N(s) = N(t)$ (`AxKhtoKKh`).
- For all $s \in W$, $[s] \in N(s)$ (`AxKtoKh`).
- For all $s \in W$ and $X \subseteq W$, if $X \in N(s)$ then $Y = \{t \mid [t] \subseteq X\} \in N(s)$ (`AxKhtoKhK`)
- For all $s \in W$ and $X, Y \subseteq W$, if $X \in N(s)$, Y is definable, and $Y \in N(x)$ for all $x \in X$, we will have $Y \in N(s)$ (`AxKhKh`).

Further directions

FUTURE DIRECTIONS

- Multi-agent knowing how: e.g., one-step coalition knowledge-how with distributed knowledge and abilities: [Naumov and Tao TARK17, AAAI17]
- Goal-maintaining [Naumov and Tao AAMAS17]
- Knowingly doing [Broersen JPL2011]
- Commonly knowing how
- Comparison with various semantics of epistemic ATL
- Characterization theorems
- Logical omniscience of knowing how
- Update of *knowing how*
- Epistemic planning [Li, Yu, Wang JLC18]