EPISTEMIC LOGIC (VIII)

Beyond "knowing that": Introduction

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Knowledge is not only expressed in terms of "knowing that":

- I *know whether* the claim is true.
- I *know what* your password is.
- I *know how* to go to the hotel.
- I *know why* he was late.
- I *know who* proved this theorem.

Hits (in millions) returned by google:

X	that	whether	what	how	who	why
"know X"	574	28	592	490	112	113
"knows X"	50.7	0.51	61.4	86.3	8.48	3.55

BEYOND "KNOWING THAT"

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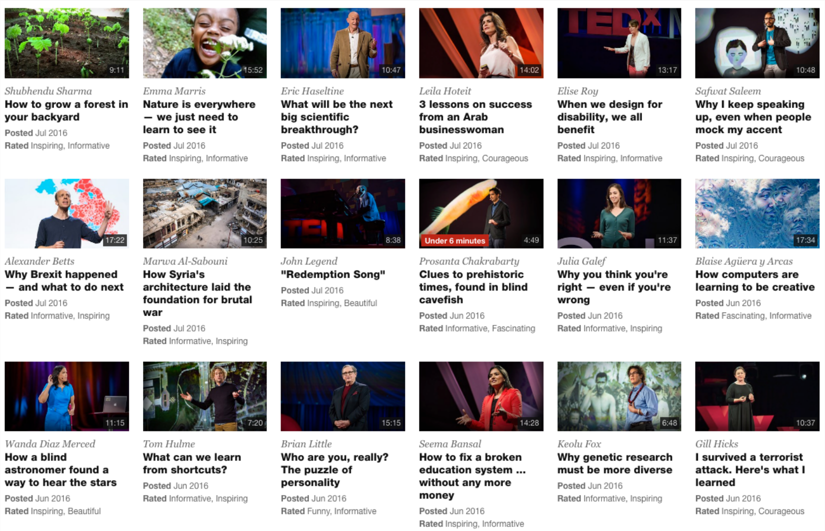
Linguistically: factivity, exhaustivity, concealed questions

Philosophically: reducible to "knowledge-that"?

Logically: how to reason about "knowing-wh"?

Computationally: efficient representation and reasoning

WE INDEED WANT TO KNOW WHY / HOW / WHAT....



It helps to go back to the starting point of epistemic logic.

"knowing who" was discussed by Hintikka (1962) in terms of first-order modal logic: $\exists x \mathcal{K}(Mary \approx x)$, i.e., knowing the answer of the embedded question.

Hintikka used epistemic logic to understand questions. E.g, consider the question Q : "Who murdered Mary?"

- The *presupposition* of Q is $\mathcal{K}\exists x M(x, Mary)$.
- The *desideratum* of Q is $\exists x \mathcal{K} M(x, Mary)$.
- One possible answer to Q is $M(John, Mary)$.
- *Conclusiveness* of the answer requires $\exists x \mathcal{K}(John \approx x)$.
- Conclusive answers realize the desideratum ($\mathcal{K}\exists x$ to $\exists x \mathcal{K}$).

In *Meaning and Necessity* (1947), Carnap remarked:

> *Any system of modal logic without quantification is of interest only as a basis for a wider system including quantification. If such a wider system were found to be impossible, logicians would probably abandon modal logic entirely.*

However, it seems that history went exactly the other way around.

MANY THINGS CAN BE DONE IN FIRST-ORDER MODAL LOGIC

First-order modal logic is infamous for:

- issues in the semantics
- *quantifying-in* and substitution
- ambiguity: *de re* vs. *de dicto*
- incompleteness
- lack of Craig's interpolation
- undecidablility (hard to find useful decidable fragments)
- ....

At the same time, propositional modal logic is too successful...

In the latest *Handbook of Epistemic Logic*, there is hardly anything about first-order epistemic logic.

A slightly out-dated survey in Gochet and Gribomont (2006)

Mostly application-driven (not an exhaustive list):

- about games: Kaneko and Nagashima (1996)
- about cryptographic knowledge: Cohen and Dam (2007)
- about security protocols: Belardinelli and Lomuscio (2011)
- (un)decidability: Wolter (2000), Sturm et al (2000)
- *de dicto* vs. *de re*: distinction Corsi and Orlandelli (2011)
- second-order epistemic logic: Belardinelli and van der Hoek (2015, 2016)
- ...

WHY IS PROPOSITIONAL MODAL LOGIC WELL-BEHAVED?

The secret of propositional modal logic:

- natural language, intuitive semantics, useful models.
- balance expressive power vs. complexity
- tame the quantifier by a guard, e.g., $\Box p : \forall x(wRx \rightarrow P(x))$.

Instead of trying to tame the infamous full quantified epistemic logic, we ...

- take a know-wh construction as a single modality, e.g., pack $\exists x K_i(Mary \approx x)$ into $Kwho_i Mary$;
- give some intuitive semantics for certain subtypes/interpretations of knowing-wh;
- axiomatize logics with (combinations of) new operators;
- dynamify those logics with new updates of knowledge;
- add new group notions, and dynamic or temporal aspects.
- automate the inferences;
- (probably) come back to philosophy and linguistics with new insights and questions.

the (potential) advantages of modal logics of "knowing-wh"

- Natural and succinct to express the desired properties;
- Limited expressive power and moderate complexity (secret of success of modal logic);
- Capture the essence of the relevant reasoning by axioms;
- Stay technically neutral in some philosophical debates;
- Formal notion of consistency of knowledge bases;

We have seen the "bundles":

- Temporal logics
- Neighbourhood semantics

BEYOND KNOWING THAT: (TECHNICAL) DIFFICULTIES

- not *normal*:
  - $\nvdash \mathcal{K}w(p \to q) \land \mathcal{K}w\, p \to Kw\, q$
  - $\nvdash \mathcal{K}h\varphi \land \mathcal{K}h\psi \to \mathcal{K}h(\varphi \land \psi)$
  - $\vdash \varphi \nRightarrow \vdash \mathcal{K}y\varphi$
- not strictly weaker: $\vdash Kw\varphi \leftrightarrow Kw\neg\varphi$;
- combinations of quantifiers and modalities: $\exists x\mathcal{K}\varphi(x)$;
- the axioms depend on the special schema of φ essentially;
- weak language vs. rich model: hard to axiomatize;
- fragments of FO/SO-modal language;

SOME PEOPLE INVOLVED SO FAR

- Jie Fan, Yanjun Li, Tszyuen Lau, Shihao Xiong, Yifeng Ding, Tao Gu, Chao Xu, Xingchi Su, Jixin Liu, Zhouhang Zhou ...;
- Hans van Ditmarsch, Malvin Gattinger, Jan van Eijck, Alexandru Baltag, Andreas Herzig, Raul Fervari, Thomas Studer, Pavel Naumov, Jia Tao, Valentin Goranko, Fernando Velázquez-Quesada, Jeremy Seligman...

- Knowing whether: [Fan, W.& van Ditmarsch: AiML14, RSL15] [Fan & vD: ICLA15, JANCL16], [Fan 17]
- Knowing what: [W. & Fan: IJCAI13, AiML14][Gu & W. AiML16], [Baltag, AiML16] [van Eijck, Gattinger, W. ICLA17]
- Knowing how: [W. LORI15], [W. Synthese17], [Li, W. ICLA17][Herzig, Fervari, Li, W. IJCAI17], [Fervari, Velázquez-Quesada, W. SR17][Naumov & Tao TARK17...]
- Knowing why: [Xu, W., Studer 18]
- Knowing who: [W., Seligman: AiML18]
- Special column in *Studies in Logic* by Fan, Li, Ding.

How to distinguish the work in this line and other related work in the literature?

Whether it uses a single modality for know-wh, instead of breaking it down into quantifiers, normal modalities, questions, predicates and so on

wh-word	bundle	connection	key ref
whether	$\mathcal{K}\varphi \vee \mathcal{K}\neg\varphi$	non-contingency logic	[FWvD14,15
what	$\exists x\mathcal{K}(\varphi \to x \approx c)$	weakly aggregative logic	[WF13,14]
how	$\exists \pi\mathcal{K}[\pi]\varphi$	game logic, ATL	[Wang15,17
why	$\exists t\mathcal{K}(t : \varphi)$	justification logic	[XWS18]

We obtained complete axiomatizations, characterizations of expressive power, and decidability ...

Along the way, we also understand better why neighbourhood semantics works for many philosophical logic.

EXAMPLE: KNOWING HOW [FERVARI, HERZIG, LI, W. IJCAI17]

TAUT	all axioms of propositional logic	MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
DISTK	$\mathcal{K}p \land \mathcal{K}(p \to q) \to \mathcal{K}q$	NECK	$\dfrac{\varphi}{\mathcal{K}\varphi}$
T	$\mathcal{K}p \to p$	EQREPKh	$\dfrac{\varphi \to \psi}{\mathcal{K}h\varphi \to \mathcal{K}h\psi}$
4	$\mathcal{K}p \to \mathcal{K}\mathcal{K}p$	SUB	$\dfrac{\varphi(p)}{\varphi[\psi/p]}$
5	$\neg\mathcal{K}p \to \mathcal{K}\neg\mathcal{K}p$		
AxKtoKh	$\mathcal{K}p \to \mathcal{K}hp$		
AxKhtoKKh	$\mathcal{K}hp \to \mathcal{K}\mathcal{K}hp$		
AxKhtoKhK	$\mathcal{K}\mathcal{K}hp \to \mathcal{K}h\mathcal{K}p$		
AxKhKh	$\mathcal{K}h\mathcal{K}hp \to \mathcal{K}hp$		
AxKhbot	$\neg\mathcal{K}h\bot$		

CONNECTIONS TO EXISTING LOGICS AND LINGUISTIC THEORIES

Classification by question words:

- Knowing whether: non-contingency logic, ignorance logic
- Knowing what: weakly aggregative logic, dependence logic
- Knowing how: game Logic, alternating temporal logic
- Knowing why: quantified justification Logic

Classification by logical forms:

- *Mention-some*: e.g., *knowing how/why*... $\exists x \mathcal{K} \varphi(x)$
- *Mention-all* (strongly exhaustive reading): e.g., I *know who* came to the party... $\forall x(\mathcal{K}\varphi(x) \vee \mathcal{K}\neg\varphi(x))$
- In-between: *know-value* $\exists x(\mathcal{K}\, c \approx x) \leftrightarrow \forall x(\mathcal{K}\, c \approx x \vee \mathcal{K}\, c \not\approx x)$

(Routine) research questions:

- Model theory, proof theory, computational complexity
- Group knowledge
- Logical omniscience
- Natural dynamics
- Applications

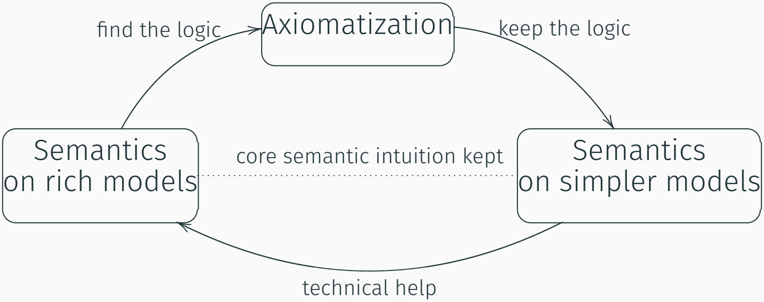
New questions:

- Interactions of different knowledge expressions
- Simplification of semantics

SIMPLIFY THE SEMANTICS WHILE KEEPING THE LOGIC

Common difficulties: weak language vs. rich semantics

To restore the balance between the language and model:



Disadvantages from a linguistic point of view:

- Compositionality
- Uniformity
- Expressivity

Disadvantages in terms of knowledge representation:

- Propositional epistemic logic is not really about the *content* of knowledge!

A question: how to explain the decidability of those logics?

What we are after:

- Expressive enough: covering the essence of those non-standard epistemic logics
- Not too much: sharing most good properties of propositional modal logic

Uniformity, compositionality, expressvity, computability: we want a predicate modal framework like the propositional modal logic

Inspired by the concrete know-wh logics, we introduce the bundle modalities into the predicate modal language:

- pack $\exists x \mathcal{K}$ into a *bundle* modality (mention-some)
- pack $\forall x \mathcal{K} w$ into a *bundle* modality (mention-all)

You can also come up with your favourite bundles.

We obtain some nice and powerful fragments of first-order modal logic.

Example: epistemic language of mention-some [W. TARK17]:

$$\varphi ::= P\overline{x} \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x \mathcal{K} \varphi$$

$\exists x \mathcal{K} \varphi$: I know some thing such that φ

- "I know a theorem of which I do not know any proof": $\exists x \mathcal{K} \neg \exists y \mathcal{K} Prove(y, x)$
- "i knows a country which j knows its capital": $\exists x \mathcal{K}_i \exists y \mathcal{K}_j Capital(y, x)$

Simply putting a decidable fragment of first-order logic plus a modality does not work at all.

Language	Decidability	Ref
P^1	undecidable	[Kripke 62]
x, y, p, P^1	undecidable	[Gabbay 93]
$x, y, \Box_i$, single P^1	undecidable	[Rybakov & Shkatov 17]

The decidable fragments are rare (only one x in $\Box$). Most of the propositional know-wh logics are in the one variable fragment.

Language	Decidability	Ref
single x	decidable	[Segerberg 73]
$x, y/P^1/\text{GF}, \Box_i(x)$	decidable	[Wolter & Zakharyaschev 01]

WHAT ABOUT OUR BUNDLED FRAGMENTS?

Good news!

- $\exists\Box$ fragment is decidable over both increasing and constant domain models! $\forall x\Diamond$ weakens the power of $\forall$!
- A satisfiable $\exists\Box$ formula has a *finite tree* model.
- We have a tableau method for satisfiability of MLMS
- Satisfiability checking of $\exists\Box$ fragment is PSPACE-complete (exactly as the complexity of propositional model logic)
- Even you allow both $\exists\Box$ and $\forall\Box$ bundles, it is still decidable over increasing domain models.

Note that we do not need to restrict the arity of the predicates or the number of variable occurrences at all.

The meaning of the world is the separation of wish and fact.

— Gödel

- $\exists\Box$ fragment is undecidable over S5 models.
- $\forall\Box$ fragment with two unary predicates is undecidable over constant domain models.

It is not as robust as propositional modal logic: we are at the edge of first-order expressivity.