EPISTEMIC LOGIC V

Dynamic Epistemic Logic (A)

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Dynamic Epistemic Logic: background

Public Announcement Logic

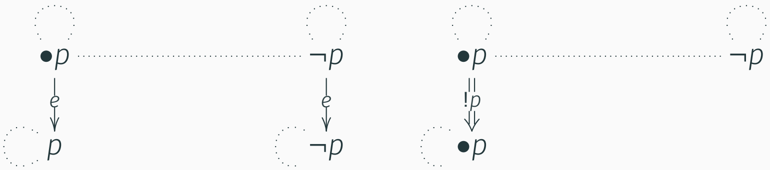
Two basic questions to be answered

Dynamic Epistemic Logic: background

Epistemic Temporal Logic (**ETL**) and *Dynamic* Epistemic Logic (**DEL**)

	language	**model**	**semantics**
ETL	time+K	temporal+epistemic	Kripke-like
DEL	K+events	epistemic	Kripke+*dynamic*

$\neg Kp \wedge [e]\, Kp$ $\qquad\qquad$ $\neg Kp \wedge [!p]Kp$



DEL handles *how* is the knowledge updated.

[Stalnaker, 1978] on *assertion*:

- Its content is *dependent* on its context.
- It *modifies* the context.

The ideas of discourse representation theory, dynamic logic and the above points together inspired the invention of *dynamic semantics* [Groenendijk and Stokhof, 1991] and *update semantics* [Veltman, 1996]:

The meaning of a sentence is identified with its *context change potential*.

(Compare it with truth conditional semantics: knowing the meaning of a sentence is knowing when it is true)

One step further:

The meaning of a communicative event is the *change* it brings to the epistemic states of the participants in the discourse.

- [Gerbrandy and Groeneveld, 1997] combined the ideas of [Veltman, 1996] and [Fagin et al., 1995]: dynamic epistemic semantics for announcements.
- [Gerbrandy, 1999] developed the idea further. Some ILLC students rediscovered [Plaza, 1989] in which the public announcement logic (PAL) was proposed and studied in depth.
- [Baltag et al., 1998] proposed the dynamic epistemic logic with action model updates.

From Web of Science database:

Overview books:

- Dynamic Epistemic Logic [van Ditmarsch et al., 2007]
- Logical Dynamics of Information and Interaction [van Benthem, 2011]

DISCIPLINE

From `Springer Link`

Computer Science	384
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Do we really understand thoroughly what we are doing? What is *Dynamic Epistemic Logic* as a field?

In searching for the answer, let us go back to the basics.

We will focus on axiomatizations:

- It helps us to understand the semantics-driven logics better.
- It helps to compare with related approaches.

Public Announcement Logic

PUBLIC ANNOUNCEMENT LOGIC (PAL)

The language of *Public Announcement Logic (*PAL*)*:

$$\phi \quad ::= \quad p \mid \neg\phi \mid (\phi \wedge \phi) \mid \mathsf{K}_i\phi \mid [\phi]\phi \ \text{(also write } [!\phi]\phi\text{)}$$

We define $\langle\psi\rangle\phi$ as $\neg[\psi]\neg\phi$.

It is interpreted on (S5) Kripke models $\mathcal{M} = (S, \{\rightarrow_i\}_{i\in I}, V)$:

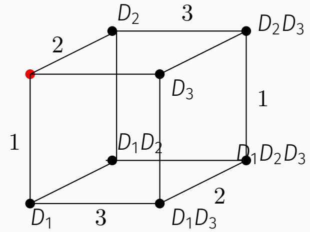
$$\boxed{\begin{array}{rcl} \mathcal{M}, s \vDash \mathsf{K}_i\psi & \Leftrightarrow & \forall t : s \rightarrow_i t \implies \mathcal{M}, t \vDash \psi \\ \mathcal{M}, s \vDash [\psi]\phi & \Leftrightarrow & \mathcal{M}, s \vDash \psi \text{ implies } \mathcal{M}|_\psi, s \vDash \phi \end{array}}$$

where $\mathcal{M}|_\psi = (S', \{\rightarrow'_i \mid i \in \mathrm{I}\}, V')$ such that: $S' = \{s \mid \mathcal{M}, s \vDash \psi\}$, $\rightarrow'_i = \rightarrow_i |_{S'\times S'}$ and $V'(p) = V(p) \cap S'$.

$$\begin{tikzcd} s_1 : \{p\} \arrow[loop above, "1"] \arrow[r, leftrightarrow, "1"] & s_2 : \{\} \arrow[loop above, "1"] \end{tikzcd} \qquad [p] \Longrightarrow \qquad \begin{tikzcd} s_1 : \{p\} \arrow[loop above, "1"] \end{tikzcd}$$

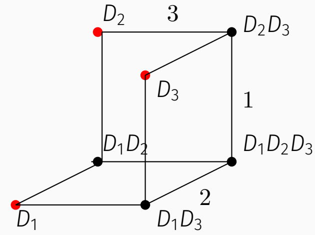
$$\mathcal{M}, s_1 \vDash \neg\mathsf{K}_1 p \wedge [p]\mathsf{K}_1 p$$

- Out of n children, $k \geq 1$ got mud on their faces while playing.
- They can see whether other kids are dirty, but there is no mirror for them to discover whether they are dirty themselves.
- Then father walks in and states: “At least one of you is dirty!” Then he requests “If you know you are dirty, step forward now.”
- If nobody steps forward, he repeats his request: “If you now know you are dirty, step forward now.”
- After exactly k requests to step forward, the k dirty children suddenly do so (assuming they are honest and perfect reasoners).



"At least one of you is dirty!"

Announcement: $\psi = D_1 \vee D_2 \vee D_3$



The classic modal logic questions:

- Do we have a complete axiomatization?
- Do we have complete axiomatizations w.r.t. certain classes of frames ?
- Do the axioms and rules of a normal modality also hold for $[\psi]$?
- Is PAL invariant under bisimulation or other equivalence notions?
- Does it have finite model property?
- Is it decidable?
- How is its definability over models and frames?
- What is the relationship between PAL and modal (epistemic) logic?
- Is it translatable into first-order logic?

Try to get a feeling of the semantics of PAL by checking the validity of the following formula schemas and rules.

- $\langle\phi\rangle\psi \to [\phi]\psi$, $\langle\phi\rangle\psi \to \phi$, $\langle\phi\rangle\psi \leftrightarrow (\phi \wedge [\phi]\psi)$
- $[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$, $[\psi](\phi \to \chi) \leftrightarrow ([\psi]\phi \to [\psi]\chi)$
- $[\psi]p \leftrightarrow (\psi \to p)$, $[\psi]\neg\phi \leftrightarrow (\psi \to \neg\phi)$ ($\times$), $[\psi]\neg\phi \leftrightarrow \neg[\psi]\phi$ ($\times$), $[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
- $\dfrac{\phi}{[\psi]\phi}$, $\dfrac{\phi(p)}{\phi(\psi)}$ ($\times$), $\dfrac{\phi \leftrightarrow \psi}{[\phi]\chi \leftrightarrow [\psi]\chi}$, $\dfrac{\phi \leftrightarrow \psi}{[\chi]\phi \leftrightarrow [\chi]\psi}$
- $[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i(\psi \to [\psi]\phi))$, $[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i[\psi]\phi)$
- $[\psi][\chi]\phi \leftrightarrow [\psi \wedge \chi]\phi$ ($\times$), $[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
- $[\psi]\mathsf{K}_i\psi$ ($\times$)

BASIC SYSTEM PA: AXIOMS AND RULES

Different proof systems were proposed in the literature which share the following axiom schemas and rules.

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\mathsf{K}_i(\phi \to \psi) \to (\mathsf{K}_i\phi \to \mathsf{K}_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
!K	$[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i[\psi]\phi)$
Rules	
NECK	$\dfrac{\phi}{\mathsf{K}_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$

No uniform substitution!

Axiom Schemas	
DIST!	$[\psi](\phi \to \chi) \to ([\psi]\phi \to [\psi]\chi)$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \wedge [\psi]\chi]\phi$
Rules	
NEC!	$\dfrac{\phi}{[\psi]\phi}$
RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$

Axiom Schemas	
`!ATOM`	$[\psi]p \leftrightarrow (\psi \rightarrow p)$
`!NEG`	$[\psi]\neg\phi \leftrightarrow (\psi \rightarrow \neg[\psi]\phi)$
`!CON`	$[\psi](\phi \wedge \chi) \leftrightarrow ([\psi]\phi \wedge [\psi]\chi)$
`!K`	$[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \rightarrow \mathsf{K}_i[\psi]\phi)$

Semantic update as syntactic relativization:

$$\mathcal{M}|_{\psi}, s \vDash \phi \iff \mathcal{M}, s \vDash (\phi)^{\psi}$$

Proposition

All the above axiom schemas and rules are sound w.r.t the standard PAL *semantics.*

Theorem ([Plaza, 1989])

PAL *is equally expressive as basic modal logic.*

$$
\begin{array}{lcllcl}
t(p) & = & p & t([\psi]p) & = & t(\psi \to p) \\
t(\neg\phi) & = & \neg t(\phi) & t([\psi]\neg\phi) & = & t(\psi \to \neg[\psi]\phi) \\
t(\phi_1 \wedge \phi_2) & = & t(\phi_1) \wedge t(\phi_2) & t([\psi](\phi_1 \wedge \phi_2)) & = & t([\psi]\phi_1 \wedge [\psi]\phi_2) \\
t(\mathsf{K}_i\phi) & = & \mathsf{K}_i t(\phi) & t([\psi]\mathsf{K}_i\phi) & = & t(\psi \to \mathsf{K}_i[\psi]\phi) \\
 & & & t([\psi][\chi]\phi) & = & t([\psi]t([\chi]\phi))
\end{array}
$$

We can obtain another translation t' by revising t: just replace the last item by $t'([\psi][\chi]\phi) = t'([\psi \wedge [\psi]\chi]\phi)$

Intuitively, the translation "pushes" the $[\cdot]$ modality through the formula to the inner part. How to prove that the translation indeed produces $[\cdot]$-free formulas?

Definition (Complexity of PAL formulas)

$$\begin{array}{lcl}
c(\top) & = & 1 \\
c(p) & = & 1 \\
c(\neg\phi) & = & 1 + c(\phi) \\
c(\phi_1 \wedge \phi_2) & = & 1 + c(\phi_1) + c(\phi_2) \\
c(\mathsf{K}_i\phi) & = & 1 + c(\phi) \\
c([\psi]\phi) & = & (5 + c(\psi)) \cdot c(\phi)
\end{array}$$

We can show that:

$$
\begin{array}{lcl}
c(\phi) > c(\psi) & & \text{If } \psi \text{ is a proper subformula of } \phi \\
c([\psi]\top) & > & c(\psi \to \top) \\
c([\psi]p) & > & c(\psi \to p) \\
c([\psi]\neg\phi) & > & c(\psi \to \neg[\psi]\phi) \\
c([\psi](\phi_1 \wedge \phi_2)) & > & c([\psi]\phi_1 \wedge [\psi]\phi_2) \\
c([\psi]K_i\phi) & > & c(\psi \to K_i[\psi]\phi) \\
c([\psi][\chi]\phi) & > & c([\psi \wedge [\psi]\chi]\phi) \\
c([\psi][\chi]\phi) & > & c([\psi]t([\chi]\phi))
\end{array}
$$

We can prove by induction on the **complexity** of ϕ that (cf. DEL book Lemma 7.22, 7.23):

Proposition

$t(\phi)$ and $t'(\phi)$ are $[\cdot]$-free.

We can show that:

Proposition

$\models \phi \leftrightarrow t(\phi)$ *and* $\models \phi \leftrightarrow t'(\phi)$

Is $t(\phi) = t'(\phi)$?

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\mathsf{K}_i(\phi \to \psi) \to (\mathsf{K}_i\phi \to \mathsf{K}_i\psi)$
!ATOM	$[\psi]p \leftrightarrow (\psi \to p)$
!NEG	$[\psi]\neg\phi \leftrightarrow (\psi \to \neg[\psi]\phi)$
!CON	$[\psi](\phi \land \chi) \leftrightarrow ([\psi]\phi \land [\psi]\chi)$
!K	$[\psi]\mathsf{K}_i\phi \leftrightarrow (\psi \to \mathsf{K}_i[\psi]\phi)$
Rules	
NECK	$\dfrac{\phi}{\mathsf{K}_i\phi}$
MP	$\dfrac{\phi, \phi \to \psi}{\psi}$
Your choice	
RE	$\dfrac{\phi \leftrightarrow \chi}{\psi \leftrightarrow \psi[\chi/\phi]}$
!COM	$[\psi][\chi]\phi \leftrightarrow [\psi \land [\psi]\chi]\phi$

Completeness is proved via reduction and the completeness of basic modal logic K:
$\vDash \phi \implies \vDash t(\phi) \overset{\text{comp. of K}}{\implies} \vdash_{\mathsf{K}} t(\phi) \implies \vdash_{\mathsf{PA}^+} t(\phi) \overset{\mathit{Rd.Axioms}}{\implies} \vdash_{\mathsf{PA}^+} \phi$ We can mimic t and t' in proof systems stronger than PA.

Proposition

$\vdash_{PA+\mathrm{RE}} \phi \leftrightarrow t(\phi)$ *and* $\vdash_{PA+\mathtt{!COM}} \phi \leftrightarrow t'(\phi)$

Theorem ([Plaza, 1989])

PA+RE is complete w.r.t. the standard semantics of PAL.

Theorem (cf. e.g., [van Ditmarsch et al., 2007])

PA+!COM *is complete w.r.t. the standard semantics of* PAL.

Now we can answer most of the following questions:

- * Do we have a complete axiomatization?
- * Do we have complete axiomatizations w.r.t. other classes of frames?
- * Do the axioms and rules for K also hold for $[\psi]$?
- * Is PAL invariant under bisimulation?
- * Is it translatable into first-order logic?
- * Does it have finite model property?
- * Is it decidable?
- * How is its definability power (over models and frames)?
- *What if we add announcement operators on propositional logic?

Theorem ([Lutz, 2006])

PAL *is exponentially more succinct than modal logic on arbitrary models.*

$\phi_0 = \top$ and $\phi_{i+1} = \langle\langle\phi_i\rangle\Diamond_1\top\rangle\Diamond_2\top$.

Theorem ([French et al., 2011])

PAL *is exponentially more succinct than modal logic on S5 models if there are more than 3 agents.*

The reduction technique turns out to be extremely useful in many applications and thus dominates the field of DEL.

- Logic is more than it appears!
- Update-closeness may be considered as a desired property of a logic: it shows the logic has enough pre-encoding power [van Benthem et al., 2006].
- Compositional analysis of post-conditions.
- The orthodox programme of DEL:
  static logic+dynamic operators+reduction
- Also good for lazy guys to have “results”...

Two basic questions to be answered

In some published papers, PA and its variants are mentioned as complete systems. Is PA really complete?

Unfortunately, PA and many of its “close friends” are not complete, and in some cases the flaws cannot be fixed.

Can we give meaningful axiomatizations without those reduction axioms and the reduction proof method?

Yes, we can!

We will give a general axiomatization method inspired by Epistemic Temporal Logic. It will tell us what exactly is assumed in DEL.

Baltag, A., Moss, L., and Solecki, S. (1998).
The logic of public announcements, common knowledge, and private suspicions.
In *Proceedings of TARK '98*, pages 43–56. Morgan Kaufmann Publishers Inc.

Fagin, R., Halpern, J., Moses, Y., and Vardi, M. (1995).
Reasoning about knowledge.
MIT Press, Cambridge, MA, USA.

French, T., van der Hoek, W., Iliev, P., and Kooi, B. (2011).
Succinctness of epistemic languages.
In *IJCAI*, pages 881–886.

Gerbrandy, J. (1999).
Bisimulations on Planet Kripke.
PhD thesis, University of Amsterdam.

Gerbrandy, J. and Groeneveld, W. (1997).
Reasoning about information change.
Journal of Logic, Language and Information, 6(2):147–169.

Groenendijk, J. and Stokhof, M. (1991).
Dynamic predicate logic.
Linguistics and Philosophy, 14(1):39 – 100.

Lutz, C. (2006).
Complexity and succinctness of public announcement logic.
In *Proceedings of AAMAS '06*, pages 137–143, New York, NY, USA. ACM.

Plaza, J. A. (1989).
Logics of public communications.

In Emrich, M. L., Pfeifer, M. S., Hadzikadic, M., and Ras, Z. W., editors, *Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems*, pages 201–216.

Stalnaker, R. (1978).
Assertion.
In Cole, P., editor, *Syntax and Semantics*, volume 9. New York Academic Press.

van Benthem, J. (2011).
Logical Dynamics of Information and Interaction.
Cambridge University Press.

van Benthem, J., van Eijck, J., and Kooi, B. (2006).
Logics of communication and change.
Information and Computation, 204(11):1620–1662.

van Ditmarsch, H., van der Hoek, W., and Kooi, B. (2007).

Dynamic Epistemic Logic.
(Synthese Library). Springer, 1st edition.

Veltman, F. (1996).
Defaults in update semantics.
Journal of Philosophical Logic, 25(3):221–261.