On Merely Expressive Devices

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Merely expressive device:

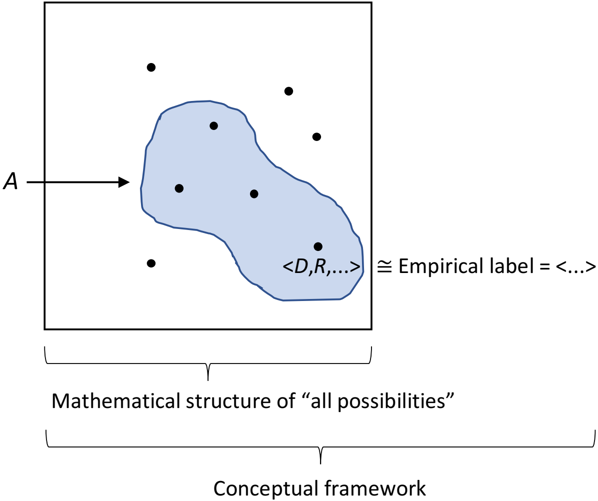
- a linguistic part of a sentence A that contributes to the proposition $[A]$ expressed by A, without contributing to the truth conditions of $[A]$.

Paradigm case: logical operators (e.g., $\vee$) are merely expressive devices.

They help us to express a proposition or to “grasp a thought” (e.g., $[A \vee B]$) without representing anything in the “world”.

(Compare Wittgenstein in the *Tractatus*: “Mein Grundgedanke ist, dass die ‘logischen Konstanten’ nicht vertreten”.)

I will reconstruct such devices in a special kind of conceptual framework: a merely expressive device in a sentence *A* contributes mathematical structure by which the set [*A*] (of mathematical "possibilities") expressed by *A* is determined. But that mathematical structure is not involved when determining whether [*A*] is true by comparing it to an intended interpretation (a "label") .



I will work out the details of such an intensional conceptual-framework semantics for merely expressive devices and draw some philosophical conclusions from it.

Instead of stating a definition of ‘conceptual framework’, I will give examples. (But it would be possible to state a fairly general definition.)

Plan:

1. Example 1: Logical Constants
2. Example 2: Stipulative Definitions and Analyticity
3. Example 3: Metaphysical Necessity
4. Conclusions and Prospects

Logical Constants

Let us build a conceptual framework. We start with a mathematical structure:

- Let W (the set of "worlds") be the set of all triples w of the form

  $$\langle D, M_1, M_2 \rangle$$

  where D is a finite non-empty initial segment of the natural numbers, and where M_1, M_2 are subsets of D.

Then we add a "label structure" that is meant to be represented:

- $label = \langle Humans, Men, Married \rangle$, where *Humans* is the set of humans, *Men* is the set of men, and *Married* is the set of all married humans (at t).

Conceptual framework:

- $F = \langle W, label \rangle$.

Definition

For all worlds $w = \langle D, M_1, M_2 \rangle$ over $F = \langle W, label \rangle$:

w is *actual-in-F* iff there is a function $l\colon D \to Humans$ over F that is a structure-preserving map between w and *label*, that is:

- l is bijective;
- for all m in D: $m \in M_1$ iff $l(m) \in Men$;
- for all m in D: $m \in M_2$ iff $l(m) \in Married$.

As we will see, there is more than one world that is actual-in-F, but all of them are pairwise isomorphic.

Call any subset of W that is closed under isomorphism a *proposition* (in F). (Compare: measurement theory.)

Call a proposition *true-in-F* iff it includes a world that is actual-in-F. (Compare: non-statement view of theories in philosophy of science.)

For instance (at an early point in time):

- $\mathit{Humans} = \{\mathit{Adam}, \mathit{Eve}\}$, $\mathit{Men} = \{\mathit{Adam}\}$, $\mathit{Married} = \{\mathit{Adam}, \mathit{Eve}\}$.
- $w_1 = \langle\{1,2\},\{1\},\{1,2\}\rangle$ is actual-in-F.
- $w_2 = \langle\{1,2\},\{2\},\{1,2\}\rangle$ is actual-in-F. ($w_2 \cong w_1$.)
- $w_3 = \langle\{1,2\},\{1\},\{1\}\rangle$ is not actual-in-F.
- $w_4 = \langle\{1,2\},\{2\},\{2\}\rangle$ is not actual-in-F. ($w_4 \cong w_3$.)
- $\{w_1, w_2\}$ is true-in-F. $\{w_3, w_4\}$ is not true-in-F.
- $\{\langle D, X, Y\rangle \mid X \neq \varnothing\}$ is true-in-F.
- W is true-in-F.

The framework does not constrain *label* (except for D being finite & non-empty). All “combinatorial possibilities” of what *label* might be like are realized in W. Therefore, by math alone, W is true-in-F: there *must* be an actual world in F.

In contrast, the truth of $\{w_1, w_2\}$ is due to F *and* the “facts”, as $\{w_1, w_2\} \neq W$!

Now add language: interpret a first-order language $\mathcal{L}$ with unary predicates *Man*, *Married* by the framework F, such that at any world $w = \langle D, M_1, M_2 \rangle$:

- D is the range of quantifiers at w,
- the interpretation of *Man* at w is M_1,
- the interpretation of *Married* at w is M_2.

In this way, one can define for all A in $\mathcal{L}$:

- A is true-at-w-in-F iff ... (standard semantic rules; w serves as model).
- A expresses proposition $[A]_F$ iff $[A]_F = \{w \in W \,|\, A \text{ is true-at-}w\text{-in-}F\}$.
- A is true-in-F iff $[A]_F$ is true-in-F.

E.g., in the Adam-Eve example it follows:

- $\exists x Man(x)$ is true-in-F.
- $[\exists x Man(x)]_F \neq W$.
- $\exists x Man(x) \vee \neg \exists x Man(x)$ is true-at-w-in-F at all w in W.
- $[\exists x Man(x) \vee \neg \exists x Man(x)]_F = W$.

In F, e.g., the 'M_1'-coordinate of a world is meant to represent *Men* (its label), which figures in the truth conditions for propositions.

If one introduces the notion of concept (as in intensional semantics), one might also say:

In F, the concept of *being a man* is meant to represent *Men* (its label), which figures in the truth conditions for propositions.

With $\mathcal{L}$ added, one might also say:

In F, the predicate *Man* expresses the concept of *being a man*, which represents *Men* (its label), which figures in the truth conditions for propositions.

That is: in F, the predicate *Man* contributes (i) to the expression of propositions *and* (via the concept it expresses) (ii) to the truth conditions for propositions.

The predicate *Man* is *not* a merely expressive device!

How about logical concepts/operators, say, *disjunction*?

If one introduces the notion of logical concept (as in intensional semantics), one might say:

In F, the concept of *disjunction* does not represent anything (any label) that would figure in the truth conditions for propositions.

With $\mathcal{L}$ added, one might also say:

In F, the operator $\vee$ expresses the concept of *disjunction*, which does not represent anything (any label) that would figure in the truth conditions for propositions.

That is: in F, the operator $\vee$ contributes (i) to the expression of propositions but *not* (conceptually) (ii) to the truth conditions for propositions.

While it holds that $[A \vee B]_F = [A]_F \cup [B]_F$, it is not the case that $\cup$ is compared to any label in the definitions of actuality or truth in F.

Upshot:

In F, logical concepts do not represent.

In F, logical operators are merely expressive devices. (One might say: they express concepts but do not represent properties “out there”.)

I do not take this to be an odd feature of F but rather a good model-theoretic reconstruction of how logical operators work semantically in natural language.

As the tradition would have said: logical constants are *syncategorematic*.

Stipulative Definitions and Analyticity

Let us expand the framework. We start with a mathematical structure again:

- Let W' (the set of "worlds") be the set of all quadruples w of the form

$$\langle D, M_1, M_2, B \rangle$$

  where D is a finite non-empty initial segment of the natural numbers, where M_1, M_2 are subsets of D, and where

$$B = (D \setminus M_2) \cap M_1 .$$

Then we add the label structure meant to be represented:

- $label' = \langle Humans, Men, Married, * \rangle$, where *Humans* is the set of humans, *Men* is the set of men, and *Married* is the set of all married humans. ('$*$' merely signals that the 'B'-coordinate is not labeled!)

Conceptual framework:

- $F' = \langle W', label' \rangle$.

Definition

For all worlds $w = \langle D, M_1, M_2, B \rangle$ over $F' = \langle W', label' \rangle$:

w is actual in F' iff there is a labeling function l: $D \rightarrow Humans$ over F' that is a structure-preserving map between w and $label'$, that is:

- l is bijective;
- for all m in D: $m \in M_1$ iff $l(m) \in Men$;
- for all m in D: $m \in M_2$ iff $l(m) \in Married$.

(No '*B*'!)

Reformulate all previous definitions as definitions for F' (*actuality, truth*,...).

The framework does not constrain *label* (except for D being finite & non-empty). By math alone, there must be an actual world in F'.

Note that this would not be so if '*B*' were labeled!

Now add language: interpret a first-order language $\mathcal{L}_{Bachelor}$ with predicates *Man*, *Married*, *Bachelor* by the framework F' analogously as before.

Reformulate all previous definitions for sentences analogously as before (*truth at world*, *expressing*, *truth*).

E.g., as before, it follows in the Adam-Eve case:

- $\exists x Man(x)$ is true-in-F'.
- $[\exists x Man(x)]_{F'} \neq W'$.
- $\exists x Man(x) \vee \neg \exists x Man(x)$ is true-at-w-in-F' at all w in W'.
- $[\exists x Man(x) \vee \neg \exists x Man(x)]_{F'} = W'$.

In addition, it follows that the sentence

$$\forall x(Bachelor(x) \leftrightarrow \neg Married(x) \wedge Man(x))$$

also expresses the set of all worlds over the framework F'.

The sentence is true-in-F' by math alone (and the definition of F')!

If we introduced the notion of concept again (as in intensional semantics), we might say:

In F', the concept of *being a bachelor* does not represent anything (any label) that would figure in the truth conditions for propositions.

With $\mathcal{L}_{Bachelor}$ added, we might also say:

In F', the predicate *Bachelor* expresses the concept of *being a bachelor*, which does not represent anything (any label) that would figure in the truth conditions for propositions.

That is: in F', the predicate *Bachelor* contributes (i) to the expression of propositions but *not* (conceptually) (ii) to the truth conditions for propositions.

While it holds that, e.g., $[\exists x Bachelor(x)]_{F'} = [\exists x(\neg Married(x) \wedge Man(x))]_{F'}$, it is not the case that $(D \setminus M_2) \cap M_1$ is compared to any label in the definitions of actuality or truth.

Upshot:

In F', the defined concept of *being a bachelor* does not represent.

In F', the predicate *Bachelor* is a merely expressive device. (One might say: it expresses a concept but does not represent a property "out there".)

I do not take this to be a odd feature of F' but rather a good model-theoretic reconstruction of how stipulative definitions work semantically.

As the tradition would have said: stipulative definitions are true by convention.

Since the meaning of *Bachelor* in F' is given by the meanings of *Married* and *Man*, the predicate *Bachelor* does not itself contribute to truth conditions of propositions, whereas the primitives *Married* and *Man* do so in F'.

More generally: define A to be analytic-in-F' iff $[A]_{F'} = W'$.

Hence, e.g., all logical truths in $\mathcal{L}_{Bachelor}$ are analytic-in-F', as is, e.g., $\forall x(Bachelor(x) \rightarrow \neg Married(x) \wedge Man(x))$.

Metaphysical Necessity

Now we build a very different framework (but presupposing analogous definitions of actuality, truth,... as before). The mathematical structure is:

- Let D be the set $\{1, 2\}$.
- Let W (the set of "worlds") be the set of all quintuples of the form

  $$\langle D, 1, 2, M, C \rangle$$

  where M and C are binary relations on D, such that C is reflexive, symmetric, transitive, and C is a congruence relation with respect to M.

  (It follows, e.g., that there are exactly two worlds in W for which C is total.)
- Let R be the uniquely defined binary accessibility relation on W, such that for all $w = \langle D, 1, 2, M, C \rangle$, $w' = \langle D, 1, 2, M', C' \rangle$ in W, it holds:

  $$R(w, w') \text{ iff } C = C'.$$

This is the label structure that is meant to be represented:

- $label = \langle \{ms, es\}, ms, es, MovePattern, Id, *\rangle$, where:
  - *ms*: last body to be seen in the sky before sunrise in the period between the beginning of the year to mid-March.
  - *es*: first body to be seen in the sky after sunset in the period between mid-August to the end of the year.
  - *MovePattern*: set of all ordered pairs $\langle d, d'\rangle$ of objects in $\{ms, es\}$ the movement pattern of which is such that periodically there is a stage of appearance of d in the morning sky without any appearance of d' in the evening sky, followed by a stage of appearance of d' in the evening sky without an appearance of d in the morning sky, followed by a stage of appearance of d in the morning sky again without an appearance of d' in the evening sky, and so on (with breaks in between the stages, and as observed from the earth).
  - *Id*: set of all ordered pairs $\langle d, d'\rangle$ of objects in $\{ms, es\}$ for which it holds that $d = d'$.
  - '$*$' merely signals that the '*R*'-component is not labeled.

Framework:

- $F = \langle W, R, label \rangle$.

Definition

For all worlds $w = \langle D, 1, 2, M, C \rangle$ over $F = \langle W, R, label \rangle$:

w is actual-in-F iff there is a labeling function l: $D \to \{ms, es\}$ over F that is a structure-preserving map between w and *label*, that is:

- $l(1) = ms$;
- $l(2) = es$;
- for all m, n in D: $\langle m, n \rangle \in M$ iff $\langle l(m), l(n) \rangle \in MovePattern$;
- for all m, n in D: $\langle m, n \rangle \in C$ iff $\langle l(m), l(n) \rangle \in Id$.

(No 'R'! The accessibility relation does not figure in the truth condition of propositions.)

The framework does not impose any constraints on the labeled components. By math alone, there must be an actual world in the framework.

It follows that there is just one world that is actual-in-F, that is,

$$w_1 = \langle D, 1, 2, \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}, \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}\rangle.$$

Now consider the following propositions:

- $A = \{w = \langle D, 1, 2, M, C\rangle \in W \mid \langle 1,2\rangle \in M\}$
  ("the morning star and the evening star instantiate the periodic moving pattern").
- $B = \{w = \langle D, 1, 2, M, C\rangle \in W \mid \langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle \in C\}$
  ("the morning star is identical to the evening star").
- $NB = \{w \in W \mid$ for all w' in W: if $R(w, w')$ then B is true-at-w'-in-$F\}$
  ("it is necessary that the morning star is identical to the evening star").

A, B, NB ($= B$) are true-in-F, since they include w_1 as a member.
The truth of A, B, NB is due to F *and* the "facts", since $A, B, NB \neq W$!

In contrast, $(W \setminus B) \cup NB$ is the set of all worlds: it is true-in-F just due to F.

Now add language again: interpret a modal first-order language with individual constants a (for *ms*) and b (for *es*), predicates *Moving-Pattern* and $=$, and the necessity operator $\Box$, over the framework F (as in possible worlds semantics).

The following three sentences express the propositions A, B, NB, respectively, and hence they are true-in-F but not analytic-in-F:

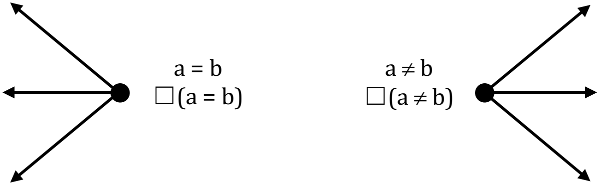
- $\textit{Moving-Pattern}(a, b)$
- $a = b$
- $\Box(a = b)$.

The sentence

- $a = b \rightarrow \Box(a = b)$

expresses the set of *all* worlds over the framework F: it is analytic-in-F.

Upshot:



In *F*, the concept *N* does not represent.

In *F*, the operator □ is a merely expressive device. (One might say: it expresses a concept but does not represent a property "out there".)

I do not take this to be an odd feature of *F* but rather a good model-theoretic reconstruction of how metaphysical operators work semantically.

□ does *not* express analyticity: $\square(a = b)$ is true-in-*F*, but $a = b$ isn't analytic-in-*F*. Those worlds are metaphysically possible that the *actual* world "sees" via *R*.

Finally, let us add some epistemic structure to drive home the Kripkean point. . .

Let *Pr* rationally reconstruct an astronomist's degree of belief function over F:

- *Pr* assigns to the singleton set of

  $$w_1 = \langle D, 1, 2, \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}, \{\langle 1,1\rangle, \langle 1,2\rangle, \langle 2,1\rangle, \langle 2,2\rangle\}\rangle$$

  a rational degree of belief of $\frac{8}{1000}$.
  (ms and es fit the periodic moving pattern, and $ms = es$.)

- *Pr* assigns to the singleton set of

  $$w_2 = \langle D, 1, 2, \{\langle 1,2\rangle\}, \{\langle 1,1\rangle, \langle 2,2\rangle\}\rangle$$

  a rational degree of belief of $\frac{2}{1000}$.
  (ms and es fit the periodic moving pattern, but $ms \neq es$.)

- For simplicity, let *Pr* assign to the singleton set of

  $$w_3 = \langle D, 1, 2, \varnothing, \{\langle 1,1\rangle, \langle 2,2\rangle\}\rangle$$

  a rational degree of belief of $\frac{99}{100}$.
  (ms and es do not fit the periodic moving pattern, and $ms \neq es$.)

Since $Pr\{w_3\} = \frac{99}{100}$, the astronomist initially regards it as highly likely that *ms* and *es* do not fit the periodic moving pattern, and that they are not identical.

One other hand, observing that *A* is going to support *B* relative to *Pr*:

- $Pr(B|A) = \frac{Pr(B \cap A)}{Pr(A)} = \frac{Pr(\{w_1\})}{Pr(\{w_1, w_2\})} = \frac{\frac{8}{1000}}{\frac{10}{1000}} = \frac{8}{10} > \frac{8}{1000} = Pr(B)$.

Thus, if the astronomist is to find out that morning star and evening star fit the moving pattern (*A*, i.e., morning star only appearing in the morning when evening star does not appear in the evening, and *vice versa*), then this makes it likely for her that they are identical (*B*). (Her degrees of belief could be so due to previous encounters with periodically appearing/disappearing phenomena.)

Since $NB = B$, it follows that *A* also supports *NB* relative to *Pr*.
(The same holds also for *NNB* and so forth.)

For the astronomer, the periodic moving pattern constitutes empirical evidence for the necessary identity of the morning star and the evening star (in *F*).

Her belief in $\Box(a = b)$ is a posteriori!

Conclusions and Prospects

We have constructed three frameworks for merely expressive linguistic devices.

These frameworks are successful rational reconstructions of existing thought and language:

- Their properties are reasonably similar to thought/language prior to reconstruction.
- Using the frameworks clarifies, precisifies, and systematizes pre-theoretic thought/language. ($\approx$ Carnap's linguistic frameworks)
- In each framework, the underlying set of worlds is a mathematical construction and is true by math alone.
  No empirical evidence is required to support its truth. (Carnap's "Empiricism, Semantics and Ontology": framework choice is pragmatic.)
  Otherwise, the frameworks would not be *conceptual* but *theories*.
- In each of them, the respective merely expressive devices are practically useful by (i) enabling us to express propositions or (ii) making the expression of propositions more convenient.

Remarks on logical constants:

- The framework of Example 1 rationally reconstructs the meaning of (classical) logical operators, such as '$\vee$'.

  '$\vee$' is framework-relative and does not represent a property "out there".

- There is a plurality of such frameworks available, many of which may be useful for various kinds of purposes.

  This includes frameworks for non-classical logic: e.g., the intuitionistic '$\vee$' can be shown to contribute *differently* to propositions than classical '$\vee$', without intuitionistic logic imposing any constraints on the "world" either.

  (Compare: Carnap's Tolerance Principle in the *Logical Syntax*.)

- Accordingly, the choice of a logical system should not be likened to a scientific inference-to-the-best-explanation (contra Williamson, Sher).

Remarks on stipulative definitions and analyticity:

- As far as constructed frameworks such as our Example 2 are concerned, *truth in virtue of meaning* is perfectly clear, precise, and unproblematic.

  $\forall x(\mathit{Bachelor}(x) \rightarrow \neg \mathit{Married}(x) \wedge \mathit{Man}(x))$ being analytic-in-F' does not reflect an "intrinsic feature" of the sentence but rather how bodies of information are "organized" in the framework (efficiently for a purpose).

  Compare: What is used as the definition of 'continuous function' in one textbook on the calculus might be a theorem in another, and *vice versa*!

- Quine's famous attack against Carnap on analyticity was misdirected: Carnap aimed to *construct* linguistic frameworks that come with *constructed* notions of analyticity and do not have factual import.

- Boghossian's (1996) "*What could it possibly mean to say that the truth of a statement is fixed exclusively by its meaning and not by the facts? [...] I cannot see how a good answer might be framed in terms of meaning, or convention*" is addressed within our second example framework.

Remarks on metaphysical necessity:

- The framework of Example 3 rationally reconstructs Kripke's (1980) claim that 'morning star = evening star' is necessary, synthetic, and a posteriori.

  It also shows that this is compatible with $\Box$ being framework-relative and not representing a property "out there".

- Perhaps metaphysics might be rationally reconstructed as the part of philosophy that (i) (re-)constructs non-representing parts of conceptual frameworks, (ii) and then recommends us to speak from within them?

  If the central terms and operators of metaphysics can be reconstructed as merely expressive devices, metaphysics would nicely fit the rational constructionism proposal that I presented yesterday.

General remarks:

- In similar ways, one might introduce conceptual frameworks for other merely expressive devices, such as a disquotationalist truth predicate, the counterfactual ‘if-then’, (structuralist accounts of) theoretical terms in science, abstraction operators in neo-logicism,...

- Perhaps mathematical symbols may themselves be viewed as merely expressive devices?

  Using mathematical symbols in scientific statements might merely help us express propositions about the “world”, without contributing to the truth conditions of these propositions.

  This would be a kind of logicism about mathematics: mathematics would merely structure “thought”, without imposing constraints on the “world”.

- Some of the tenets of the logical empiricists might be vindicated *to the extent to which they were dealing with merely expressive devices*.

Thank you!